

7

Interpreting Linear Equations & Models

On the SAT, you will encounter linear model questions that are a direct extension of our previous chapters on lines and word problems. You'll be asked to figure out the meaning of certain variables and numbers in these models within a real world context.

EXAMPLE 1:

$$V(t) = 240,000 - 5,000t$$

The given function V models the value, in dollars, of a home from 2006 to 2015, where t is the number of years after 2006. Which of the following is the best interpretation of 240,000 in this context?

- A) The value, in dollars, of the home in 2006
- B) The value, in dollars, of the home in 2015
- C) The average value, in dollars, of the home from 2006 to 2015
- D) The increase, in dollars, in the value of the home from 2006 to 2015

We're given a linear function in $y = mx + b$ form. The y -intercept b will typically designate an initial value, the value when $x = 0$. In this case, the y -intercept is 240,000 and it describes the value of the home when $T = 0$, zero years after 2006, which, of course, is 2006. Answer (A).

EXAMPLE 2:

$$V(t) = 240,000 - 5,000t$$

The given function V models the value, in dollars, of a home from 2006 to 2015, where t is the number of years after 2006. Which of the following is the best interpretation of 5,000 in this context?

- A) The number of homes sold each year
- B) The yearly decrease, in dollars, in the value of the home
- C) The difference, in dollars, between the value of the home in 2006 and in 2015
- D) The yearly decrease, in dollars, in the value of the home per square foot

This time, we're asked to interpret 5,000. Again, we should look at the given function as a linear model expressed in $y = mx + b$ form. In this case, the slope m is $-5,000$. Since the slope, by definition, is a rate of change, the increase or decrease in y (value of the home) for each unit increase in x (each year), the number 5,000 can best be interpreted as the amount, in dollars, the value of the home decreases by each year. Answer

(B).

It's important that you don't get tricked into choosing a rate that looks right but ultimately doesn't fit the context set by the variable definitions—in this case, the definitions of t and V . Answer A is wrong because we're not dealing with the number of homes sold; we're dealing with the value of a home. Answer D is wrong because the numbers in the equation aren't on a per-square-foot basis. Always be aware of how the variables in the problem are defined.

EXAMPLE 3: Evan spent a total of \$275 on x cups of regular coffee and y cups of specialty coffee. The equation $3x + 5y = 275$ represents this situation. Which of the following is the best interpretation of 5 in this context?

- A) The number of cups of regular coffee Evan bought
- B) The number of cups of specialty coffee Evan bought
- C) The price, in dollars, of one cup of regular coffee
- D) The price, in dollars, of one cup of specialty coffee

Although this is an easy question, we're going to use this example to illustrate a technique that's helpful in dealing with these types of questions. The technique involves looking at the units of each term in the equation.

Here, we're given a linear equation of the form $ax + by = c$. The 275 on the right side of the equation represents the 275 dollars that were spent in total. Since the left side of the equation sums to this 275 dollars, the terms $3x$ and $5y$ must each be in dollars for the equation to make sense:

$$3x \text{ dollars} + 5y \text{ dollars} = 275 \text{ dollars}$$

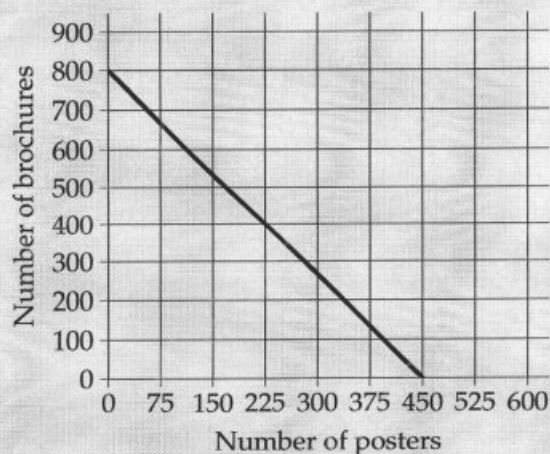
Therefore, $5y$ represents a dollar amount. Since y is defined as the number of cups of specialty coffee, the only way the units work out is if the coefficient 5 represents a dollar amount per cup of specialty coffee.

$$5 \frac{\text{dollars}}{\text{cup of specialty coffee}} \times y \text{ cups of specialty coffee} = 5y \text{ dollars}$$

Therefore, the only reasonable interpretation of 5 is that it's the price, in dollars, per cup of specialty coffee.

Answer (D).

Of course, it's not hard to get to this answer intuitively. It's easy to see that $5y$ must be the amount Evan spent on specialty coffee, so 5 must be the price per cup. There's no need to write out all the units if this is an easy question for you, but this technique may serve you on more difficult questions.

EXAMPLE 4:

A marketing agency is given a \$7,200 budget to print posters and brochures for a client. The line shown models the possible combinations of posters and brochures that the agency could print. According to the model, what is the cost of printing each poster?

- A) \$9
- B) \$12
- C) \$16
- D) \$25

When dealing with a linear model that represents possible combinations, the key is to look at either the x -intercept or y -intercept. Since we want to find the cost of printing each poster, we should look at the x -intercept—450 posters can be printed when no brochures are printed. Therefore, the cost of printing each poster is $\frac{7,200}{450} = \$16$. Answer **(C)**.

EXAMPLE 5:

$$h(x) = \frac{4x + 6}{5}$$

The function h models the maximum height, in inches, of a plant that was grown using x grams of fertilizer. Based on this model, using 1 additional gram of fertilizer in growing the plant would increase its maximum height by how many inches?

What we're essentially looking for is the change in h (maximum height) for every 1 unit increase in x (fertilizer)—in other words, the slope. To find the slope, we can put the equation into $y = mx + b$ form by splitting up the fraction: $h(x) = \frac{4}{5}x + \frac{6}{5}$. Now we can easily see that the slope is $\frac{4}{5}$ or **.8**. To summarize, the slope of 0.8 indicates that each additional gram of fertilizer increases the maximum height of the plant by 0.8 inches.

EXAMPLE 6:

$$h(x) = \frac{4x + 6}{5}$$

The function h models the maximum height, in inches, of a plant that was grown using x grams of fertilizer. Based on this model, how many more grams of fertilizer should be used in growing the plant to raise the maximum height of the plant by exactly 1 inch?

Because this question is asking for the change in x for every 1 unit increase in h (the reverse of the previous example), we need to rearrange the equation so that we have x in terms of h .

$$\frac{4x + 6}{5} = h$$

$$4x + 6 = 5h$$

$$4x = 5h - 6$$

$$x = \frac{5}{4}h - \frac{3}{2}$$

The slope of this resulting equation indicates that x must increase by $\frac{5}{4}$, or $\boxed{1.25}$, when we want h to increase by 1.

A shortcut for this type of question is to take the reciprocal of the slope of the original equation: the reciprocal of $\frac{4}{5}$ is $\frac{5}{4}$, or 1.25.

EXAMPLE 7: A can of soda is put into a freezer. The function $T(m) = 65 - 6m$ models the temperature of the soda, in degrees Fahrenheit, m minutes after the can has been put into the freezer. According to the model, what is the decrease in the temperature of the soda, in degrees Fahrenheit, for every 5 minutes the can is left in the freezer?

The slope of -6 represents the change in the temperature of the soda for every 1 minute the can is left in the freezer. So for every 5 minutes left in the freezer, the temperature of the soda decreases by $5 \times 6 = \boxed{30}$ degrees Fahrenheit.

EXERCISE 1: Answers for this exercise start on page 338.

1

The function $w(x) = 100 - 3x$ gives the water level, in feet, in a large aquarium x days after the aquarium was last refilled. Which statement is the best interpretation of the slope of the graph of $y = w(x)$ in the xy -plane in this context?

- A) The water level decreases by 3 feet each day.
- B) The water level increases by 3 feet each day.
- C) The water level decreases by 100 feet each day.
- D) The water level increases by 100 feet each day.

2

Annie collects educational puzzle sets for her classroom. She has B beginner-level puzzle sets and E intermediate-level puzzle sets. There are a total of 1,400 puzzle pieces in all her puzzle sets. The equation $36B + 64E = 1,400$ represents this situation. An intermediate-level puzzle set has how many more puzzle pieces than a beginner-level puzzle set?

3

$$f(x) = 24 + 6x$$

The given function f models the weight, in ounces, of a serving tray containing x empty glasses. According to the model, how many ounces does each empty glass weigh?

- A) 4
- B) 6
- C) 12
- D) 24

4

A total of 175 ounces of flour was used to make bagels and muffins. The equation $4.8x + 1.5y = 175$ represents this situation, where x is the number of bagels made and y is the number of muffins made. According to the equation, how many ounces of flour are used to make each bagel?

5

A local football team consists of 11 starting players and additional backup players. At the start of the football season, a total of 159 jerseys were purchased for the team, and each player received 3 jerseys. The equation $3(x + 11) = 159$ represents this situation. Which of the following is the best interpretation of the expression $x + 11$ in this context?

- A) The number of backup players on the team
- B) The number of starting players on the team
- C) The number of players on the team
- D) The number of jerseys purchased for the starting players on the team

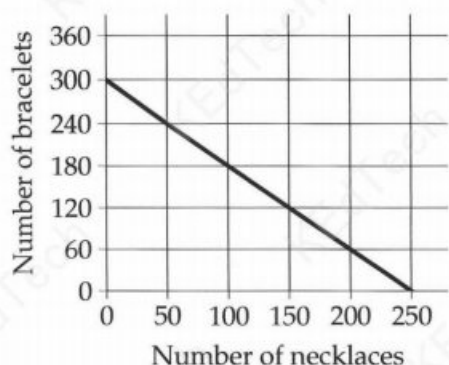
6

$$y = 177 + 42x$$

The equation gives the number of juniper trees y in a certain forest x years after 2010. What is the best interpretation of $(x, y) = (5, 387)$ in this context?

- A) The number of juniper trees in the forest increased by 387 in 2015.
- B) The number of juniper trees in the forest increased by 387 from 2010 to 2015.
- C) There were 387 juniper trees in the forest in 2005.
- D) There were 387 juniper trees in the forest in 2015.

7



The line shown models the possible combinations of the number of bracelets and necklaces a certain company with 25 jewelers can handcraft in a month, based on the constant rate at which the jewelers work. Based on this model, what is the maximum number of bracelets that one jeweler at this company could handcraft in a month?

- A) 10
- B) 12
- C) 21
- D) 30

8

Ivy bought an 80-ounce block of cheese. She then cut the block into x slices of cheese and y cubes of cheese. The equation $0.6x + 0.25y = 80$ represents this situation. Which is the best interpretation of $0.6x$ in this context?

- A) The amount of cheese, in ounces, in each cube
- B) The amount of cheese, in ounces, in each slice
- C) The total amount of cheese, in ounces, Ivy cut into cubes
- D) The total amount of cheese, in ounces, Ivy cut into slices

9

During a road trip, Carl used 12 gallons of fuel driving a minivan and 17 gallons of fuel driving a campervan. He traveled a total distance of 590 miles driving both vehicles. The equation $12x + 17y = 590$ represents this situation. Which is the best interpretation of y in this context?

- A) The total number of gallons of fuel Carl used driving the campervan
- B) The total number of miles Carl drove the campervan for
- C) The fuel efficiency, in miles per gallon, of the campervan
- D) The average fuel efficiency, in miles per gallon, of the minivan and the campervan

10

Of the 585 photos in a photo album, 315 are color photos and the rest are black-and-white. The equation $6p + 315 = 585$ describes this situation, where p represents the number of pages that contain black-and-white photos. What is the best interpretation of $6p$ in this context?

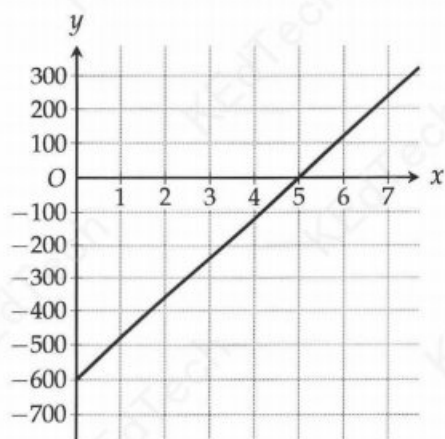
- A) There are $6p$ black-and-white photos.
- B) There are $6p$ photos on each page.
- C) There are $6p$ pages in total.
- D) There are $6p$ photos in total.

11

Stacy is playing a game in which she needs to collect a certain number of points to win. The function p defined by $p(x) = 52 - 4x$ models the number of additional points Stacy needs to win the game, where x is the number of rounds played. Which of the following is the best interpretation of $p(8) = 20$?

- A) Stacy collected a total of 20 points in the first 8 rounds.
- B) It will take Stacy 8 rounds to collect the 20 additional points she needs to win the game.
- C) After the first 8 rounds, Stacy needs to collect 20 more points to win the game.
- D) In the 8th round, Stacy collected 20 more points than she did the previous round.

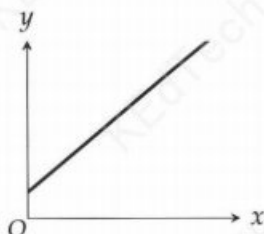
12



The graph models the daily profit y , in dollars, of a bakery after x cakes are sold. Which of the following is the best interpretation of the y -intercept of the graph in this context?

- A) The price of each cake
- B) The cost of making each cake
- C) The daily cost of running the bakery
- D) The daily cost of making the cakes that weren't able to be sold

13



A train began its route a certain distance away from a train depot and traveled in a straight path directly away from it. The line shown represents the train's distance from the depot, in miles, x hours after it began its route. What is the best interpretation of the slope of the line?

- A) The speed of the train, in miles per hour, during its route
- B) The distance, in miles, between the train depot and the train at the start of its route
- C) The total distance, in miles, the train has traveled x hours after it began its route
- D) The number of hours it took for the train to finish its route

14

$$T = 56 + 5h$$

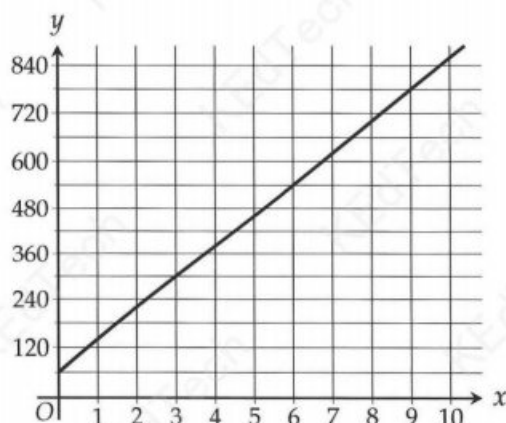
The given equation models the temperature T of a room, in degrees Fahrenheit, h hours after a heater is turned on. According to the model, the temperature of the heated room increases by how many degrees Fahrenheit every 30 minutes?

15

A recipe suggests sweetening honey tea with sugar according to the equation $s = 10 - 2h$, where s is the amount of sugar, in teaspoons, that should be added to a tea beverage containing h teaspoons of honey. What is the best interpretation of 2 in this context?

- A) For every teaspoon of honey in the beverage, two more teaspoons of sugar should be added.
- B) For every teaspoon of honey in the beverage, two fewer teaspoons of sugar should be added.
- C) For every two teaspoons of honey in the beverage, one more teaspoon of sugar should be added.
- D) For every two teaspoons of honey in the beverage, one fewer teaspoon of sugar should be added.

16



The graph represents the total time y , in minutes, a lawyer spent reviewing a case and providing x legal consultations. What is the best interpretation of the slope of the graph in this context?

- A) The lawyer spent 60 minutes reviewing the case.
- B) Each legal consultation lasted 60 minutes.
- C) The lawyer spent 80 minutes reviewing the case.
- D) Each legal consultation lasted 80 minutes.

17

$$T(x) = \frac{2x + 9}{5}$$

The function T models the time, in seconds, it takes to load a web page with x images. According to the model, by how many seconds does each image increase the load time of a web page?

18

Jamil reached the end of a 26-mile track by walking at a speed of 5 miles per hour and running at a faster speed than he walked. The equation $5t + 9(4 - t) = 26$ represents this situation, where t is the total amount of time, in hours, Jamil spent walking. Which of the following is the best interpretation of 4 in this context?

- A) The total distance, in miles, that Jamil ran
- B) The speed at which Jamil ran, in miles per hour
- C) The total number of hours Jamil spent running
- D) The total number of hours Jamil spent walking and running

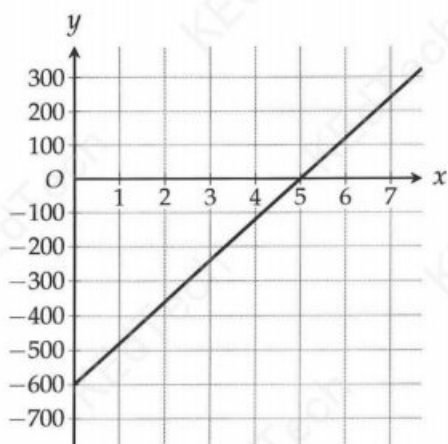
19

$$A = \frac{7}{2}(9 + x)$$

The function A gives the area, in square inches, of a trapezoid with a height of 7 inches, a shorter base of 9 inches, and a longer base of x inches. If the length of the longer base is increased by 5 inches, by how many square inches does the area of the trapezoid increase?

EXERCISE 2: Answers for this exercise start on page 339.

1



The graph models the daily profit y , in dollars, of a bakery after x cakes are sold. Which of the following is the best interpretation of the slope of the graph in this context?

- A) The price of each cake
- B) The profit generated from each cake sold
- C) The daily profit generated from all the cakes that were sold
- D) The number of cakes that need to be sold to make a daily profit of 100 dollars

2

Potassium sorbate is mixed into yogurt to increase its shelf life. The function $f(x) = 4 + 3x$ models the shelf life, in days, of 120 ounces of yogurt with x grams of potassium sorbate mixed in. Which of the following is the best interpretation of $f(4) = 16$ in this context?

- A) The shelf life of 120 ounces of yogurt with 4 grams of potassium sorbate mixed in is about 16 days.
- B) The shelf life of 120 ounces of yogurt with 16 grams of potassium sorbate mixed in is about 4 days.
- C) Mixing in 4 grams of potassium sorbate increases the shelf life of 120 ounces of yogurt by about 16 days.
- D) Mixing in 16 grams of potassium sorbate increases the shelf life of 120 ounces of yogurt by about 4 days.

3

A tank initially contained 204 gallons of water. After a hole was punched in the tank, the amount of water in the tank decreased at a constant rate each minute until 36 gallons of water remained in the tank. The equation $204 - 4.2x = 36$ represents this situation, where x is the number of minutes after the hole was punched. Which is the best interpretation of the number 4.2 in this context?

- A) The percentage decrease in the amount of water in the tank
- B) The rate at which water left the tank, in gallons per minute
- C) The number of minutes it took for every 1-gallon decrease in the amount of water in the tank
- D) The total decrease, in gallons, in the amount of water in the tank x minutes after the hole was punched

4

A seafood vendor earned \$725 for selling a pounds of fish and b pounds of shrimp. The equation $12a + 18b = 725$ represents this situation. What is the best interpretation of $18b$ in this context?

- A) The total number of shrimp the vendor sold
- B) The total weight, in pounds, of all the fish and shrimp the vendor sold
- C) The total amount, in dollars, the vendor earned for b pounds of shrimp sold
- D) The total amount, in dollars, the vendor earned for each pound of shrimp sold

5

A total of 200 dollars was spent to purchase notebooks and binders. The equation $2.50x + 3.50y = 200$ represents this situation, where x is the number of notebooks purchased and y is the number of binders purchased. What is the best interpretation of $(x, y) = (24, 40)$ in this context?

- A) If 24 notebooks were purchased, then 40 binders were purchased.
- B) If 24 notebooks were purchased, then 40 dollars was spent to purchase them.
- C) If 40 notebooks were purchased, then 24 binders were purchased.
- D) If 40 notebooks were purchased, then 24 dollars was spent to purchase them.

6

A website provides video tutorials on computer programming to its subscribers. The function $s(x) = 500 + 200x$ gives the number of subscribers when there were x videos available on the site. What is the best interpretation of 500 in this context?

- A) The site gained 500 subscribers for every additional video it made available.
- B) The site initially had 500 videos available.
- C) The site had 500 subscribers when there were no videos available.
- D) The site gained 500 subscribers for every 200 additional videos it made available.

7

An office supply store sells desks at a discount to customers who purchase at least 5 desks. For a purchase of x desks, the price per desk y , in dollars, is given by the equation $y = 500 - 5x$, where $5 \leq x \leq 30$. Which statement is the best interpretation of $(x, y) = (6, 470)$ in this context?

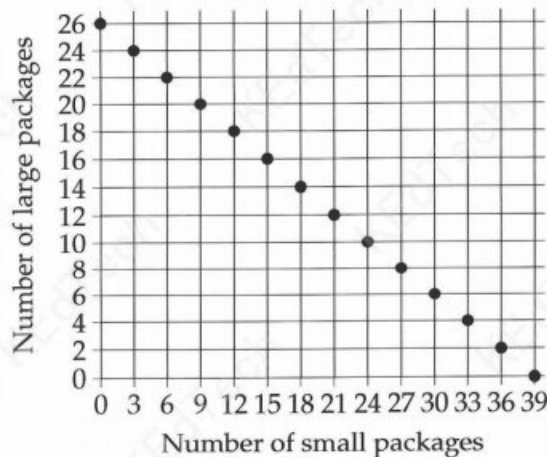
- A) A customer purchasing 6 desks will pay a total of 470 dollars.
- B) A customer purchasing 6 desks will receive a discount of 470 dollars.
- C) A customer purchasing 6 desks will pay 470 dollars for each desk.
- D) A customer purchasing 470 desks will receive a discount of 6 dollars on each desk.

8

A total of 400 patients were treated at a health clinic run by doctors and nurse practitioners. Each patient was treated by either a doctor or a nurse practitioner. Of the patients treated, 127 were treated by doctors. The equation $21n + 127 = 400$ represents this situation, where n is the number of patients treated by each nurse practitioner at the clinic. Which of the following is the best interpretation of $21n$ in this context?

- A) The number of nurse practitioners at the clinic
- B) The number of patients treated by nurse practitioners at the clinic
- C) The number of doctors at the clinic
- D) The difference between the number of patients treated by nurse practitioners at the clinic and the number of patients treated by doctors at the clinic

9



Olivia sells figurines and ships them in small and large packages. It takes her 10 minutes to prepare each small package and 15 minutes to prepare each large package. The graph represents the possible combinations of the number of small packages and the number of large packages Olivia can prepare in exactly k minutes. What is the value of k ?

- A) 270
- B) 330
- C) 390
- D) 450

10

A walking trail consists of a level path that is 7 miles long and a sloped path that is 4 miles long. It took Oscar a total of 5 hours to walk the entire trail. The equation $7x + 4y = 5$ represents this situation. Which of the following is the best interpretation of y in this context?

- A) The total number of hours it took Oscar to walk the sloped path
- B) Oscar's average speed, in miles per hour, while walking the sloped path
- C) The average number of hours it took Oscar to walk one mile of the sloped path
- D) Oscar's average speed, in miles per hour, while walking the level path

11

A party store sells premade banners and custom banners. Yesterday, the store sold a total of 80 banners, earning x dollars in revenue from premade banners and y dollars in revenue from custom banners. The equation $\frac{x}{14} + \frac{y}{25} = 80$ represents this situation. Which of the following is the best interpretation of 14 in this context?

- A) The store sold 14 premade banners yesterday.
- B) The store earns 14 dollars in revenue for each premade banner sold.
- C) The store sold 14 custom banners yesterday.
- D) The store earns 14 dollars in revenue for each custom banner sold.

12

$$h(t) = 100 - 4t$$

The given function h models the number of hours until a gallon of milk held at a temperature of t , in degrees Celsius, goes sour. Which of the following is the best interpretation of the number 4 in this context?

- A) An increase of 1°C will make a gallon of milk go sour 4 hours faster.
- B) An increase of 1°C will make 4 gallons of milk go sour 1 hour faster.
- C) An increase of 4°C will make a gallon of milk go sour 1 hour faster.
- D) An increase of 4°C will make a gallon of milk go sour 4 hours faster.

13

$$h(t) = 3.5\left(\frac{t}{50} + 2\right)$$

The given function h models the height of a tree, in inches, t days after it is planted. According to the model, about how many inches does it grow each day?

- A) 0.02
- B) 0.07
- C) 1.75
- D) 3.5

14

An antique lamp was sold at an auction. The price p of the lamp, in dollars, during the auction can be modeled by the equation $p = 900 - 10t$, where t is the number of seconds left in the auction. What is the best interpretation of 900 in this context?

- A) The starting auction price of the lamp
- B) The final auction price of the lamp
- C) The increase in the price of the lamp per second
- D) The time it took to auction off the lamp, in seconds

15

A teacher printed sticker sheets with 7 stickers on each sheet. At a school party, she gave out 2 stickers to each student who attended. The relationship between the number of students who attended the party, x , and the number of sticker sheets that the teacher printed, y , is represented by the equation $7y - 32 = 2x$. What is the best interpretation of 32 in this context?

- A) The teacher gave out a total of 32 stickers at the party.
- B) The teacher had 32 sticker sheets remaining after the party.
- C) The teacher printed 32 more stickers than she gave out at the party.
- D) The teacher printed a total of 32 sticker sheets.

16

$$y = 1.30x - 1.50$$

The given equation is used by a bank to convert x U.S. dollars into y euros. The bank performs the conversion using an exchange rate and charges a fee for the conversion. Which of the following is the best interpretation of 1.50 in this context?

- A) The bank charges 1.50 euros to do the currency exchange.
- B) The bank charges 1.50 U.S. dollars to do the currency exchange.
- C) One U.S. dollar is worth 1.50 euros.
- D) One euro is worth 1.50 U.S. dollars.

17

$$C(x) = 0.07(x - 180) + 62.6$$

The function C gives the monthly cost, in dollars, of using an international phone plan for x minutes each month. If the phone plan is used for 75 more minutes each month, by how many dollars does the monthly cost of the plan increase?

- A) 1.75
- B) 5.25
- C) 55.25
- D) 67.85

18

$$C(x) = 1.5 + 2.5x$$

The given function C models the cost, in dollars, of mailing a shipment weighing x pounds. According to the model, an increase of 10 dollars in the mailing cost corresponds to an increase of how many pounds in the weight of the shipment?

- A) 2
- B) 2.5
- C) 4
- D) 5