4

Other Types of Equation Questions

In this chapter, we'll cover more question types involving equations.

1. Matching terms

EXAMPLE 1: The expressions $(x+10)(x+b)$ and $x^2+3bx+c$, where b and c are constants, are equivalent. What is the value of c ?

We essentially have two variables and one equation. Let's expand $(x+10)(x+b)$ first and see if that takes us anywhere.

$$(x+10)(x+b) = x^2+bx+10x+10b = x^2+(b+10)x+10b$$

Notice that we grouped the x terms, bx and $10x$. You'll see why in a moment. So now we have

$$x^2+(b+10)x+10b = x^2+3bx+c$$

For both sides to be equal to each other, the corresponding terms on each side must be equal. For example, the coefficient of the x term on the left side must equal the coefficient of the x term on the right side. Let's underline the corresponding terms:

$$x^2+\underline{(b+10)x}+\underline{\underline{10b}} = x^2+\underline{3bx}+\underline{c}$$

Equating the corresponding coefficients and constants gives us

$$b+10 = 3b$$
$$10b = c$$

Solving the first equation, we get $b=5$. Plugging this result into the second equation, we get $c = 10(5) = \boxed{50}$.

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2. Infinitely many solutions

Another way that the SAT tests this "matching terms" concept is to frame the question in the context of **infinitely many solutions** for a *single* equation (we'll talk about infinitely many solutions for a *system* of equations in another chapter).

A single equation has infinitely many solutions when both sides of the equation are equivalent.

For instance,

$$3x + 6 = 3x + 6$$

has infinitely many solutions because no matter what the value of x is, the equation is always true ($x = 1$ is a solution, $x = 2$ is a solution, $x = 3$ is a solution, ...) Notice that the equation boils down to $0 = 0$, which, again, is always true.

Equations like $3x + 6 = 3x + 6$ are a bit weird because what's the point of dealing with them in the first place? Of course $3x + 6$ is equal to $3x + 6$! But keep in mind that these equations are not meant to be solved; they're meant to demonstrate the concept of infinitely many solutions. Let's take a look at how this concept might appear in an SAT question.

EXAMPLE 2:

$$a(x^2 - 2b) = 4x^2 - 12$$

In the given equation, a and b are constants. The equation has infinitely many solutions. What is the value of b ?

As in Example 1, we can expand the left side and match the terms so that both sides are equivalent. Only when both sides are equivalent does the equation have infinitely many solutions.

$$a(x^2 - 2b) = 4x^2 - 12$$
$$ax^2 - 2ab = 4x^2 - 12$$

Comparing corresponding terms, we can see that $a = 4$ and $-2ab = -12$. Now we can solve for b.

$$-2ab = -12$$
$$-2(4)b = -12$$
$$b = \frac{-12}{-8} = \boxed{1.5}$$

EXAMPLE 3:

$$kx + 3(5 - 2x) = 15$$

In the given equation, k is a constant. If the equation is true for all values of x, what is the value of k ?

This question is just another way of testing the infinitely many solutions concept. Expanding the left side of the equation, we get

$$kx + 15 - 6x = 15$$

Since the right side is just a constant of 15, we need to cancel out the x terms on the left side in order for both sides to be equivalent. It's easy to see that $k = \boxed{6}$ does the job. If it helps, you can think of the right side as having a $0x$ term. The end result is that no matter what the value of x is, 15 equals 15. Yes, I know these equations are weird, but that's how you get infinitely many solutions.

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3. No solutions

The opposite of infinitely many solutions is **no solutions**. When an equation has no solutions, there are no values of x that satisfy it. To illustrate, the equation

$$3x + 6 = 3x + 10$$

has no solutions because there is no value of x that can ever make $3x + 6$ equal to $3x + 10$. The equation itself is a contradiction. This is even more obvious if we subtract $3x$ from both sides: we're left with $6 = 10$, which is fundamentally false.

Now let's go back to the original equation, $3x + 6 = 3x + 10$, for a moment. Notice that the x terms on each side have the same coefficient of 3, but the constants of 6 and 10 are different. These are the signs of an equation with no solution.

A single equation has no solutions when both sides of the equation are equivalent EXCEPT for the constants. The constants must be different.

EXAMPLE 4:

$$3cx - 4(x + 1) = 2(x - 1)$$

In the given equation, c is a constant. The equation has no solution. What is the value of c ?

Expanding each side, we get

$$\begin{aligned} 3cx - 4(x + 1) &= 2(x - 1) \\ 3cx - 4x - 4 &= 2x - 2 \\ 3cx - 4 &= 6x - 2 \end{aligned}$$

The constants are already different, so we just need to get the coefficients of x to match. Very simply, $3c = 6$, which gives $c = \boxed{2}$.

EXAMPLE 5: How many solutions does the equation $6(x + 2) = 6x + 8$ have?

A) Zero B) Exactly one C) Exactly two D) Infinitely many

Expanding the left side, we get

$$6x + 12 = 6x + 8$$

Both sides are equivalent except for the constants, 12 and 8, so this equation must have no solution. Answer $\boxed{(A)}$. Note that subtracting $6x$ from both sides yields $12 = 8$, a contradiction that further confirms our answer.

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4. Absolute Value

The absolute value of x, denoted by $|x|$, is the distance x is from 0. In other words, absolute value makes everything positive. If the value is positive, it stays positive. If it's negative, it becomes positive.

EXAMPLE 6:

$$|x+3| = 8$$

What is one possible solution to the given equation?

Because of the absolute value, the expression inside can take on a positive or negative value, so we have two possibilities:

- $x+3 = 8$
- $x+3 = -8$

Solving these equations, we get $x = 8-3 = \boxed{5}$ and $x = -8-3 = \boxed{-11}$ as possible solutions.

5. Finding the number of solutions to various types of equations

EXAMPLE 7:

$$(2x-3)^2 = -1$$

How many distinct real solutions does the given equation have?

A) Zero B) Exactly one C) Exactly two D) Infinitely many

The square of anything can never be negative, so this equation has zero real solutions. Answer $\boxed{(A)}$. We'll talk more extensively about the number of solutions to quadratic equations in a later chapter.

EXAMPLE 8:

$$|x-10| = 0$$

How many solutions does the given equation have?

A) Zero B) One C) Two D) Three

As we saw earlier, absolute value equations typically have two solutions, one for the positive case and one for the negative case. The equation in this example, however, has a constant of zero, and because zero does not have positive and negative counterparts, the only possibility is $x-10 = 0$, which has just one solution, $x = 10$. Because there is only one solution, the answer is $\boxed{(B)}$.

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EXERCISE 1: Answers for this exercise start on page 325.

1

$$\frac{1}{4}x - 6 = px + 5$$

In the given equation, p is a constant. The equation has no solution. What is the value of p ?

2

$$|x + 20| = 8$$

What is one possible solution to the given equation?

3

$$3x + 6 = \frac{12x + r}{4}$$

In the given equation, r is a constant. The equation has infinitely many solutions. What is the value of r ?

A) 6
B) 8
C) 12
D) 24

4

$$|x - 15| = 32$$

What is the sum of the solutions to the given equation?

5

How many solutions does the equation $3(x - 2) = 6(x + 1) - 3x$ have?

A) Zero
B) Exactly one
C) Exactly two
D) Infinitely many

6

$$\frac{1}{5}x = k(x - k)$$

In the given equation, k is a constant. The equation has no solution. What is the value of k ?

7

If $|3x + 9| = 156$, what is the positive value of $x + 3$?

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8

$$6x = x - 3x(2n - 1)$$

In the given equation, n is a constant. The equation has infinitely many solutions. What is the value of n ?

A) $-\frac{2}{3}$

B) $-\frac{1}{3}$

C) $\frac{4}{3}$

D) $\frac{5}{3}$

9

How many solutions does the equation $-|x+6| = 21$ have?

A) Zero
B) Exactly one
C) Exactly two
D) More than two

10

$$2(x+k) = 2x+2$$

In the given equation, k is a constant greater than 1. How many solutions does the equation have?

A) Zero
B) Exactly one
C) Exactly two
D) Infinitely many

11

$$12x + 16nx = 20$$

In the given equation, n is a constant. The equation has no solution. What is the value of n ?

12

How many solutions does the equation $-14(x-5) = 5(14-x)$ have?

A) Zero
B) Exactly one
C) Exactly two
D) Infinitely many

13

$$(-x)^2 = 6$$

How many distinct real solutions does the given equation have?

A) Zero
B) Exactly one
C) Exactly two
D) Infinitely many

14

$$3|5-x| = 42 - 4|5-x|$$

What is the positive solution to the given equation?

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15

$$(x-4)^2 - a = x^2 + bx + b$$

In the given equation, a and b are constants. If the equation is true for all values of x, what is the value of a ?

16

$$|x| = x$$

How many solutions does the given equation have?

A) Zero
B) Exactly one
C) Exactly two
D) Infinitely many

17

The expressions $bx^2 + 11x$ and $x(4x + a) - 3x$, where a and b are constants, are equivalent. What is the value of $a + b$?

A) 12
B) 15
C) 18
D) 21

18

The expression $(ax + 5)(bx - 5)$, where a and b are constants, can be rewritten as $cx^2 + 15x - 25$, where c is a constant. What is the value of $b - a$?

19

The expression $12x^2 - mx - 20$, where m is a constant, is equivalent to $(kx - 5)(nx + 4)$, where k and n are integer constants. Which of the following must be an integer?

A) $\frac{12}{m}$
B) $\frac{12}{n}$
C) $\frac{m}{k}$
D) $\frac{m}{n}$

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EXERCISE 2: Answers for this exercise start on page 326.

1

$$ax - b = 3(2x + 1)$$

In the given equation, a and b are constants. The equation has no solution. Which of the following could be the values of a and b ?

A) $a = 2$ and $b = -3$
B) $a = 2$ and $b = 3$
C) $a = 6$ and $b = -3$
D) $a = 6$ and $b = 3$

2

$$|-8x - 12| = 34$$

Exactly how many solutions does the given equation have?

A) Zero
B) One
C) Two
D) Four

3

$$9 - 6x = -\frac{1}{11}(66x - 99)$$

How many solutions does the given equation have?

A) Zero
B) Exactly one
C) Exactly two
D) More than two

4

$$\frac{3}{2}(x + m) = mx$$

In the given equation, m is a constant. If the equation has exactly one solution, which of the following could NOT be the value of m ?

A) $-\frac{3}{2}$
B) 0
C) $\frac{1}{2}$
D) $\frac{3}{2}$

5

The expression $1.56x^2 - 0.84x - 1.08$ can be rewritten as $n(26x^2 - 14x - 18)$, where n is a constant. What is the value of n ?

6

$$x - \frac{1}{2}(3x + 8) = 2\left(2 - \frac{1}{4}x\right)$$

How many solutions are there to the given equation?

A) Zero
B) Infinitely many
C) Exactly one
D) Exactly two

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7

$$\frac{2}{3}ax + 3 = \frac{8}{3}x + 9b$$

In the given equation, a and b are constants. The equation has infinitely many solutions. What is the value of $\frac{a}{b}$?

A) $\frac{3}{4}$

B) $\frac{4}{3}$

C) 6

D) 12

8

$$3x + a(3 - 2x) = 12 - 7x$$

In the given equation, a is a constant. The equation has no solution. What is the value of a ?

A) -2

B) 2

C) 4

D) 5

9

$$2|10 - 4x| + 9 = 77$$

What is the sum of the solutions to the given equation?

10

$$\sqrt{-x} = x$$

How many distinct real solutions does the given equation have?

A) Zero

B) Exactly one

C) Exactly two

D) Infinitely many

11

$$|x - 13| = |x + 5|$$

What is the solution to the given equation?

A) -8

B) -4

C) 4

D) 8

12

$$-10x + 16jx = 0$$

In the given equation, j is a constant. The equation has infinitely many solutions. What is the value of j ?

13

$$g(x) = \left|\frac{7}{9}x - 40\right|$$

The function g is defined by the given equation. For which of the following values of a does $g(a) = a$?

A) -180

B) $\frac{27}{2}$

C) $\frac{45}{2}$

D) $\frac{360}{7}$

14

How many solutions does the equation $(x + 1)(x - 1) = x(x - 1)$ have?

A) Zero

B) One

C) Two

D) Infinitely many

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15

$$34x^2 = -68x^2$$

How many solutions does the given equation have?

A) Zero
B) Exactly one
C) Exactly two
D) Infinitely many

16

$$\frac{1}{3}(bx + c) = \frac{14}{27}x + \frac{4}{9}$$

In the given equation, b and c are constants and $c > 3$. The equation has no solution. What is the value of b ?

17

$$(2x + 3)(ax - 5) = 12x^2 + bx - 15$$

In the given equation, a and b are constants. If the equation is true for all values of x, what is the value of b ?

A) 6
B) 8
C) 10
D) 12

18

The expression $5x^5 + 6x^4 - 8x^3$ can be rewritten as $(x^3 - hx^2)(5x^2 + 10x)$, where h is a constant. What is the value of h ?

19

$$18x^2 - 8 = 2(ax + b)(ax - b)$$

In the given equation, a and b are constants. If the equation is true for all values of x, which of the following could be the value of ab ?

A) 6
B) 9
C) 12
D) 36

20

The expression $(x + a)(x + 6)$, where a is a constant, is equivalent to $x^2 + bx + c$, where b and c are positive integer constants. Which of the following must be an integer?

A) $\frac{c}{6}$
B) $\frac{b}{a}$
C) $\frac{c}{b}$
D) $\frac{6}{a}$

21

$$4x^2 + mx + 9 = (2x + n)^2$$

In the given equation, m and n are constants and $n < 0$. If the equation is true for all values of x, what is the value of $m + n$?

A) -15
B) -9
C) -3
D) 12

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