

24

Probability

Generally speaking, probability can be defined as

$$\frac{\text{Number of target outcomes}}{\text{Number of total possible outcomes}}$$

Most of the probability questions on the SAT will involve tables of data. For these table questions, probability can be more narrowly defined as

$$\frac{\text{Number in target group within group being selected from}}{\text{Number in group being selected from}}$$

EXAMPLE 1:

	Beef	Chicken
First Class	18	27
Coach	62	138

The table summarizes the distribution of dinner preferences for passengers on a particular flight. If a first class passenger is selected at random from this flight, what is the probability that the selected passenger prefers a beef dinner?

- A) $\frac{9}{40}$ B) $\frac{2}{5}$ C) $\frac{3}{5}$ D) $\frac{2}{3}$

The group we're selecting from is the group of first class passengers, of which there are $18 + 27 = 45$. The target group, which must be a subset of the group we're selecting from, consists of first class passengers who prefer a beef dinner, of which there are 18. The desired probability is then

$$\frac{\text{Number of first class passengers who prefer a beef dinner}}{\text{Number of first class passengers}} = \frac{18}{45} = \frac{2}{5}$$

Answer B.

EXAMPLE 2: The table gives the distribution of vehicles on a lot by type and year of manufacture.

	Cars	Trucks	Total
2012	173	126	299
2013	182	143	325
2014	165	109	274
Total	520	378	898

If one of these vehicles is selected at random, which of the following is closest to the probability that the vehicle is either a car manufactured in 2012 or a truck manufactured in 2014?

- A) 0.193 B) 0.314 C) 0.352 D) 0.421

In this question, we're selecting from all vehicles on the lot, a total of 898 vehicles. The target group consists of cars manufactured in 2012 and trucks manufactured in 2014, a total of $173 + 109 = 282$ vehicles. The desired probability is then

$$\frac{\text{Number of 2012 cars on the lot} + \text{Number of 2014 trucks on the lot}}{\text{Total number of vehicles on the lot}} = \frac{282}{898} \approx 0.314$$

Answer B.

EXAMPLE 3: The table shows the distribution of occupation and years of experience for workers in a trade union.

	Years of Experience					Total
	1	2	3	4	5+	
Electrician	15	45	20	30	50	160
Welder	25	35	55	10	40	165
Plumber	60	5	65	70	55	255
Carpenter	15	25	50	35	60	185
Total	115	110	190	145	205	765

One of these workers will be selected at random. What is the probability of selecting a plumber, given that the worker has at least 4 years of experience? (Express your answer as a decimal or fraction, not as a percent.)

In probability questions, the phrase "*given that*," which is just another way of saying "*assuming that*," always leads to information that narrows down the group being selected from.

In this case, it's assumed that the selected worker will be one of the ones who have at least 4 years of experience, so we're not selecting from the entire group of workers but from those who have at least 4 years of experience. The desired probability is then

$$\frac{\text{Number of plumbers who have 4+ years of experience}}{\text{Number of workers who have 4+ years of experience}} = \frac{70 + 55}{145 + 205} = \frac{125}{350} = \frac{5}{14}$$

EXERCISE 1: Answers for this exercise start on page 404.

1

A box contains a total of 80 cookies. A cookie is to be selected at random from the box. If the probability that a chocolate chip cookie will be selected is 0.45, how many chocolate chip cookies are in the box?

2

The table summarizes the distribution of color and size for 450 shirts.

	Small	Medium	Large	Total
Gray	55	80	30	165
Navy	45	70	20	135
Black	50	65	35	150
Total	150	215	85	450

If one of these shirts is selected at random, what is the probability of selecting a gray shirt? (Express your answer as a decimal or fraction, not as a percent.)

3

In a display case, there are 11 gold, 30 silver, 17 platinum, and 6 titanium rings. If one of these rings is selected at random, what is the probability of selecting a ring that is neither silver nor titanium?

- A) $\frac{17}{64}$
 B) $\frac{7}{16}$
 C) $\frac{9}{16}$
 D) $\frac{7}{9}$

4

The table shows the distribution of sunglass lenses in a box by type and shape.

	Photochromic	Polarized
Rectangular	9	7
Round	5	6

If one of these lenses is selected at random, what is the probability of selecting a round photochromic lens?

- A) $\frac{5}{9}$
 B) $\frac{5}{11}$
 C) $\frac{5}{14}$
 D) $\frac{5}{27}$

5

Color	Red	Blue	Black	White	Silver
Percent	20%	33%	10%	14%	

A car manufacturer produces cars in red, blue, black, white, and silver. The incomplete table shows the percentage of cars it produces in each color. If a car produced by the manufacturer is selected at random, what is the probability of selecting a red or silver car?

- A) 23%
 B) 33%
 C) 37%
 D) 43%

6

A fair die has 12 sides each labeled with a number from 1 through 12, with a different number on each side. Two students each roll this die once. The first student rolls a 3. What is the probability that the second student also rolls a 3?

- A) $\frac{1}{4}$
 B) $\frac{3}{4}$
 C) $\frac{1}{12}$
 D) $\frac{1}{144}$

7

The table shows the distribution by type and ink color of pens stored in a bin.

Type	Ink color	Number of pens
Ballpoint	Blue	67
	Black	112
	Red	41
	Green	93
Felt-tip	Blue	24
	Black	136
	Red	58

Based on the table, what fraction of the ballpoint pens stored in the bin have red ink?

- A) $\frac{41}{531}$
 B) $\frac{41}{313}$
 C) $\frac{41}{272}$
 D) $\frac{58}{313}$

8

A whole number greater than 10 but less than 20 will be selected at random. What is the probability that the selected number will be even?

- A) $\frac{4}{8}$
 B) $\frac{4}{9}$
 C) $\frac{6}{9}$
 D) $\frac{4}{10}$

9

	Violation Type			
	Speeding	Stop sign	Parking	Total
Truck	68	39	17	124
Car	83	51	26	160
Total	151	90	43	284

The table shows the distribution of driving violations recorded by a local police department by type and vehicle. Based on the table, what fraction of the recorded stop sign violations were committed by truck drivers?

- A) $\frac{39}{284}$
 B) $\frac{39}{124}$
 C) $\frac{39}{90}$
 D) $\frac{51}{90}$

10

	Won	Lost	Total
Underdog	10	35	45
Favorite	25	5	30
Total	35	40	75

The table shows the results of 75 baseball games played by a certain team, categorized by whether the team was considered the favorite (expected to win) in the game or the underdog (expected to lose). What fraction of the games in which the team was considered the underdog did the team win?

- A) $\frac{2}{5}$
 B) $\frac{2}{7}$
 C) $\frac{2}{9}$
 D) $\frac{2}{15}$

11

A tank contains a total of n fish, of which g are guppies. If a fish is selected at random from the tank, what is the probability of selecting a fish that is not a guppy?

- A) $\frac{n-g}{g+n}$
 B) $\frac{n}{g+n}$
 C) $1 - \frac{g}{n-g}$
 D) $1 - \frac{g}{n}$

12

	Defective	Not defective	Total
Assembly line A	300	5,700	6,000
Assembly line B	500	3,500	4,000
Total	800	9,200	10,000

A manufacturer uses two assembly lines to produce refrigerator units. The table summarizes the results of a quality control test used to check for defective units. If a refrigerator unit that was checked during this test is selected at random, what is the probability that the unit was produced by assembly line A, given that the unit is defective?

- A) $\frac{1}{20}$
 B) $\frac{3}{8}$
 C) $\frac{3}{5}$
 D) $\frac{5}{8}$

13

Data set A: 3, 5, 13, 15, 17, 17

Data set B: 5, 9, 13, 17, 21, 25

The lists give the values in data sets A and B. If a number is selected at random from data set A, what is the probability that the selected number is not a number in data set B?

- A) $\frac{1}{6}$
 B) $\frac{1}{3}$
 C) $\frac{1}{2}$
 D) $\frac{2}{3}$

14

	\$11–20	\$21–30	\$31+	Total
Cashmere	9	14	28	51
Wool	18	13	44	75
Cotton	40	15	9	64
Total	67	42	81	190

The table summarizes 190 scarves sold at a store by price and material. If a wool scarf is selected at random from this store, what is the probability that the scarf has a price of at least 21 dollars? (Express your answer as a decimal or fraction, not as a percent.)

15

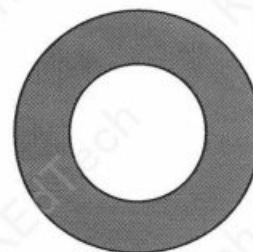
The given table summarizes the distribution of 180 beverage items by type and sugar content, in grams.

	Less than 20 g	20–30 g	Greater than 30 g	Total
Soda	17	35	43	95
Juice	22	56	7	85
Total	39	91	50	180

One of these beverage items will be selected at random. What is the probability of selecting a juice beverage, given that the beverage's sugar content is no more than 30 grams?

- A) $\frac{7}{50}$
 B) $\frac{13}{30}$
 C) $\frac{3}{5}$
 D) $\frac{78}{85}$

16



Note: Figure not drawn to scale.

In the figure shown, a circle with a radius of 4 inches has the same center as a circle with a radius of 6 inches. If a point within the figure is selected at random, what is the probability that the point is within the shaded region? (Express your answer as a decimal or fraction, not as a percent.)

17

Favorite flavor	Percent
Vanilla	28%
Chocolate	14%
Strawberry	24%
Cookies and cream	18%
Coffee	12%
Butter pecan	4%

The table shows the results of a survey in which respondents were asked to specify their favorite ice cream flavor. If a respondent is selected at random, what is the probability that the respondent's favorite flavor is cookies and cream, given that the respondent's favorite flavor is neither vanilla nor coffee?

- A) $\frac{9}{50}$
 B) $\frac{3}{10}$
 C) $\frac{3}{7}$
 D) $\frac{9}{20}$

EXERCISE 2: Answers for this exercise start on page 405.

1

The table gives the distribution of residential buildings in a neighborhood by type and year of construction.

	2005	2010	2015	Total
Single-family home	24	49	41	114
Duplex	37	28	63	128
Condominium	61	53	8	122
Total	122	130	112	364

One of these buildings will be selected at random. What is the probability of selecting a building that was constructed in 2015?

- A) $\frac{41}{364}$
 B) $\frac{4}{13}$
 C) $\frac{61}{182}$
 D) $\frac{4}{9}$

2

	In stock	Out of stock
Brand A	12	21
Brand B	18	14
Brand C	11	24

The given table summarizes the distribution of 100 items at a store by brand and availability. If one brand A item is selected at random, what is the probability that the selected item is in stock?

- A) $\frac{3}{25}$
 B) $\frac{12}{41}$
 C) $\frac{4}{11}$
 D) $\frac{4}{7}$

3

In a group of 420 nurses, some specialize in pediatrics, some specialize in cardiac care, and the rest specialize in oncology. If one of these nurses is chosen at random, the probability of choosing a nurse who specializes in cardiac care is $\frac{2}{7}$ and the probability of choosing a nurse who specializes in oncology is $\frac{15}{28}$. How many nurses in the group specialize in pediatrics?

4

There are h jars of honey, m jars of mayonnaise, and p jars of peanut butter in a cupboard. If one of these jars is selected at random, which expression represents the probability of selecting either a jar of honey or a jar of peanut butter?

- A) $\frac{h}{h+p}$
 B) $\frac{h+p}{h+p-m}$
 C) $\frac{h+p}{h+m+p}$
 D) $\frac{h+p-m}{h+m+p}$

5

	Number of soil samples	Percent of samples contaminated with glyphosate
Area 1	450	8%
Area 2	550	6%

The table summarizes the results of a research study in which soil samples collected from two areas were tested for glyphosate contamination. If one of these soil samples is selected at random, what is the probability of selecting a soil sample that is contaminated with glyphosate?

- A) 0.067
- B) 0.069
- C) 0.070
- D) 0.072

6

The table shows the results of a survey in which students at three colleges were asked whether they live on campus or off campus.

	Off campus	On campus	Total
College A	335	689	1,024
College B	124	482	606
College C	596	274	870
Total	1,055	1,445	2,500

If one of the students surveyed is selected at random, which of the following probabilities has the greatest value?

- A) The probability that the student lives off campus, given that the student attends College C.
- B) The probability that the student lives on campus, given that the student attends College C.
- C) The probability that the student attends college C, given that the student lives off campus.
- D) The probability that the student attends college C, given that the student lives on campus.

7

Scientists at Companies A and B

	Has spoken at conferences before	Has not spoken at conferences before
Company A	40	80
Company B	30	120

A national conference invites scientists from certain research companies to speak on advancements in their field. The table summarizes the prior speaking experiences of scientists at two research companies that will be represented at the conference. A scientist from company A is randomly selected to be its representative at the conference, and a scientist from company B is randomly selected to be its representative. The probability that company A's representative has spoken at conferences before is how much greater than the probability that company B's representative has spoken at conferences before?

- A) $\frac{2}{15}$
- B) $\frac{1}{4}$
- C) $\frac{3}{10}$
- D) $\frac{1}{3}$

8

On Friday, a bakery produced 700 bagels per hour for the first 3 hours of operation and 300 bagels per hour for the remaining 5 hours of operation. If a bagel produced by the bakery on Friday is selected at random, what is the probability the bagel was produced during the first 4 hours of operation?

9

A new medical test is designed to give a positive indicator to a patient who has a certain virus and a negative indicator to a patient who does not. The table summarizes the results for 1,000 patients who have taken this test.

	Negative indicator	Positive indicator	Total
Has virus	30	370	400
Does not have virus	550	50	600
Total	580	420	1,000

Based on the table, what is the probability that the test gives the wrong indicator?

- A) 5%
- B) 8%
- C) 10%
- D) 12%

10

The table summarizes the distribution of age and favorite sport for 150 members of a country club.

	Squash	Tennis	Golf	Total
30–44 years	18	10	22	50
45–59 years	11	14	25	50
60+ years	21	26	3	50
Total	50	50	50	150

One of these members will be selected at random. What is the probability of selecting a member whose favorite sport is tennis, given that the member is at least 45 years of age? (Express your answer as a decimal or fraction, not as a percent.)

11

There are a total of 500 three-digit numbers from 101 to 600. If one of these numbers is chosen at random, what is the probability that the number is less than 301? (Express your answer as a decimal or fraction, not as a percent.)

12

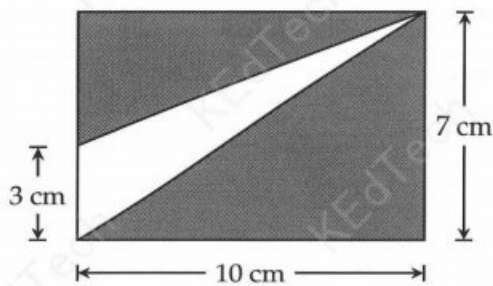
At a certain school, each student was asked whether the school's budget surplus should be spent on gym equipment or computers. The number of students who answered "computers" is 160. If a student at the school is selected at random, the probability of selecting a student who answered "gym equipment" is $\frac{1}{3}$. What is the total number of students at this school?

13

A collection of books is organized into 7 stacks of paperback books and 9 stacks of hardcover books. Each stack of paperback books consists of 4 books published in 2010 and 6 books published in 2011. Each stack of hardcover books consists of 8 books published in 2010 and 8 books published in 2011. If a book from one of these stacks is selected at random, what is the probability of selecting a paperback book, given that the book was published in 2011?

- A) $\frac{21}{107}$
- B) $\frac{7}{19}$
- C) $\frac{3}{7}$
- D) $\frac{3}{5}$

14



Note: Figure not drawn to scale.

The figure shows a rectangle and a triangle. If a point within the figure is to be selected at random, how much greater is the probability of selecting a point in the shaded region than the probability of selecting a point in the unshaded region?

- A) $\frac{4}{7}$
- B) $\frac{3}{5}$
- C) $\frac{24}{35}$
- D) $\frac{11}{14}$

15

A positive integer less than or equal to 125 is to be selected at random. What is the probability of selecting a two-digit number that is a multiple of 3? (Express your answer as a decimal or fraction, not as a percent.)

16

The table summarizes the distribution of participants in a health study by weight group.

Weight group	Percent
Less than 110 pounds	10%
110–149 pounds	25%
150–175 pounds	47%
Greater than 175 pounds	18%

If a participant in this study is selected at random, what is the probability of selecting a participant who weighs at least 150 pounds, given that the participant weighs at least 110 pounds? (Express your answer as a decimal or fraction, not as a percent.)

17

A field inhabited by plants has a side with rich soil and a side with poor soil. The table summarizes the distribution of plant species, in percent, for each side.

	Clover	Holly	Laurel	Privet	Total
Poor soil	45%	25%	20%	10%	100%
Rich soil	20%	35%	15%	30%	100%

Twice as many plants inhabit the side with rich soil as the side with poor soil. If a laurel plant is selected at random from the field, what is the probability that the laurel plant inhabits the side with rich soil?