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Radians

A radian is simply another unit of angle measure. Just as we have feet and meters for distances and pounds and kilograms for weights, we have both degrees and radians for angles.

$$\pi \text{ radians} = 180^\circ$$

If you've never used radians before, don't be put off by the π . After all, it's just a number. We could've written

$$3.14 \text{ radians} \approx 180^\circ$$

instead, but 3.14 is only an approximation. Values are typically expressed in terms of π when radians are being used.

EXAMPLE 1: An angle has a measure of 45 degrees. What is the measure of this angle, in radians?

- A) $\frac{\pi}{6}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{3}$ D) $\frac{\pi}{2}$

To convert from degrees to radians, we use the conversion factor $\frac{\pi \text{ radians}}{180^\circ}$.

$$45^\circ \times \frac{\pi \text{ radians}}{180^\circ} = \frac{\pi}{4} \text{ radians}$$

Answer **(B)**. Notice that the degree units (represented by the circle symbols) cancel out just as they should in any conversion problem.

EXAMPLE 2: An angle has a measure of $\frac{3\pi}{2}$ radians. What is the measure of the angle, in degrees?

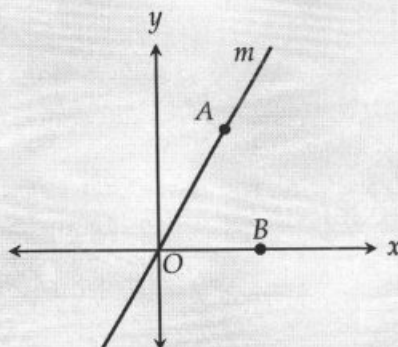
This time, we're going from radians to degrees, so we should flip the conversion factor we used in Example 1:

$$\frac{3\pi}{2} \text{ radians} \times \frac{180^\circ}{\pi \text{ radians}} = \boxed{270}^\circ$$

You might be wondering why we even need radians. Why not just stick with degrees? Is this another difference between the U.S. and the rest of the world, like it is with feet and meters? Nope. Some calculations are much simpler when the angles are in radians. For example, the length of an arc or the area of a sector can be found very easily when the central angle is in radians:

$$\text{Arc Length} = r\theta \qquad \text{Area of a Sector} = \frac{1}{2}r^2\theta$$

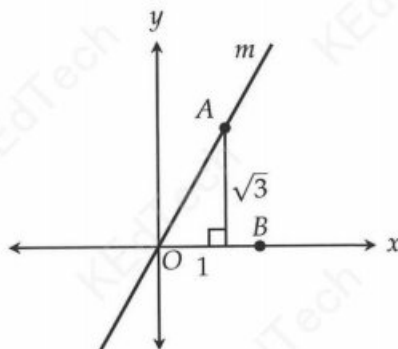
In these formulas, r is the length of the circle's radius and θ is the radian measure of the central angle. These formulas are nice to know, but you don't need to have them memorized for the SAT.

EXAMPLE 3:

In the xy -plane, point A lies on line m and point B lies on the x -axis as shown. If line m passes through the point $(0,0)$ and has a slope of $\sqrt{3}$, what is the measure, in radians, of angle AOB ?

- A) $\frac{\pi}{6}$ B) $\frac{\pi}{5}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{3}$

We can draw a line segment from point A down to the x -axis to make a right triangle. Because the slope of line m is $\sqrt{3}$, the ratio of the height of this triangle to its base is always $\sqrt{3}$ to 1 (rise over run).



In other words, this right triangle has legs whose lengths are in a ratio of $\sqrt{3}$ to 1. It must therefore be a 30–60–90 triangle, in which the angle opposite the $\sqrt{3}$ -side is 60° . The measure of angle AOB is then

$$60^\circ \times \frac{\pi \text{ radians}}{180^\circ} = \frac{\pi}{3} \text{ radians}$$

Answer (D).

EXERCISE: Answers for this exercise start on page 392.

1

An angle has a measure of $\frac{17\pi}{12}$ radians. What is the measure of the angle, in degrees?

2

An angle has a measure of 225 degrees. What is the measure of this angle, in radians?

- A) $\frac{3\pi}{4}$
- B) $\frac{7\pi}{6}$
- C) $\frac{5\pi}{4}$
- D) $\frac{3\pi}{2}$

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The measure of angle A is $\frac{5\pi}{9}$ radians. The measure of angle B is $\frac{7\pi}{36}$ radians less than the measure of angle A . What is the measure of angle B , in degrees?

- A) 35
- B) 65
- C) 100
- D) 135

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The measure of angle P is 162 degrees greater than the measure of angle R . How much greater is the measure of angle P than the measure of angle R , in radians?

- A) $\frac{9}{20}\pi$
- B) $\frac{9}{10}\pi$
- C) $\frac{10}{9}\pi$
- D) $\frac{20}{9}\pi$

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An arc of a circle has a measure of 3.4 radians. What is the measure, in degrees, of this arc, to the nearest degree? (Disregard the degree sign when entering your answer. For example, if your answer is 36° , enter 36)

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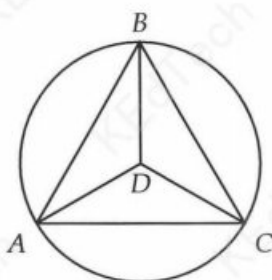
A circle has center O , and points A and B lie on the circle. The length of arc AB is $\frac{1}{5}$ of the circumference of the circle. What is the measure, in radians, of angle AOB ?

- A) $\frac{\pi}{10}$
- B) $\frac{\pi}{5}$
- C) $\frac{2\pi}{5}$
- D) $\frac{5\pi}{2}$

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An angle has a measure of 900 degrees. The number of radians in this angle can be written as $k\pi$, where k is a constant. What is the value of k ?

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In the figure shown, equilateral triangle ABC is inscribed in a circle with center D . What is the measure, in radians, of angle ADB ?

- A) $\frac{2\pi}{3}$
- B) $\frac{3\pi}{4}$
- C) $\frac{4\pi}{5}$
- D) $\frac{5\pi}{6}$

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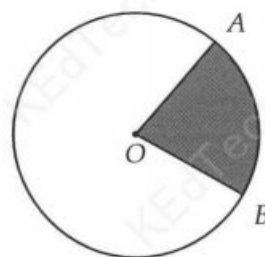
Point O is the center of a circle. The measure of arc MN on this circle is $\frac{4\pi}{3}$ radians. The length of arc MN is what fraction of the circumference of the circle?

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A circle has center O , and points X and Y lie on the circle. The measure of angle OXY is $\frac{\pi}{9}$ radians. What is the measure, in radians, of angle XOY ?

- A) $\frac{5\pi}{18}$
- B) $\frac{5\pi}{9}$
- C) $\frac{2\pi}{3}$
- D) $\frac{7\pi}{9}$

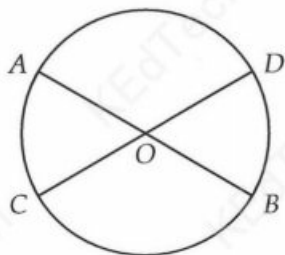
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In the figure shown, a circle has center O and a radius of 6 inches. The area of the shaded sector is 10π square inches. What is the measure, in radians, of angle AOB ?

- A) $\frac{2\pi}{5}$
- B) $\frac{4\pi}{9}$
- C) $\frac{5\pi}{9}$
- D) $\frac{5\pi}{8}$

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Note: Figure not drawn to scale.

The circle shown has center O and diameters $\overline{AB}$ and $\overline{CD}$. The length of arc AD is three times the length of arc BD . What is the measure of angle AOD , in radians?

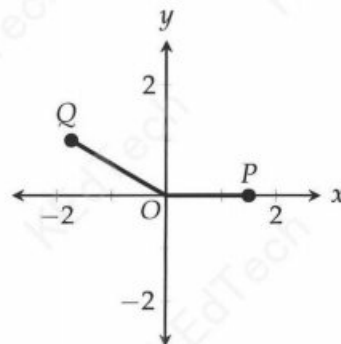
- A) $\frac{\pi}{4}$
- B) $\frac{\pi}{3}$
- C) $\frac{2\pi}{3}$
- D) $\frac{3\pi}{4}$

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In the xy -plane, segment OA has endpoints $(0, 0)$ and $(\sqrt{3}, \sqrt{3})$. What is the measure, in radians, of the acute angle formed by segment OA and the line $y = 0$?

- A) $\frac{\pi}{6}$
- B) $\frac{\pi}{4}$
- C) $\frac{\pi}{3}$
- D) $\frac{\pi}{2}$

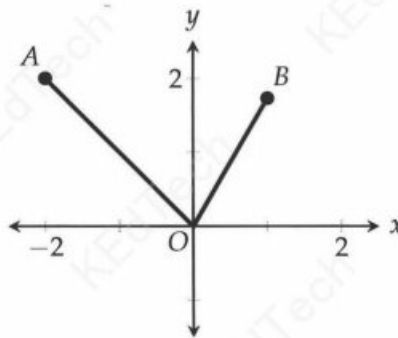
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In the figure shown, the coordinates of point Q are $(-\sqrt{3}, 1)$. What is the measure, in radians, of angle POQ ?

- A) $\frac{2\pi}{3}$
- B) $\frac{3\pi}{4}$
- C) $\frac{5\pi}{6}$
- D) $\frac{11\pi}{12}$

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In the figure shown, points A and B have coordinates $(-2, 2)$ and $(1, \sqrt{3})$, respectively. What is the measure, in radians, of angle AOB ?

- A) $\frac{3}{8}\pi$
- B) $\frac{5}{12}\pi$
- C) $\frac{4}{9}\pi$
- D) $\frac{7}{12}\pi$