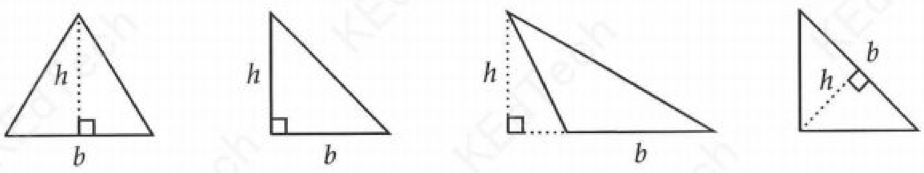
18

Triangles

Area of a Triangle

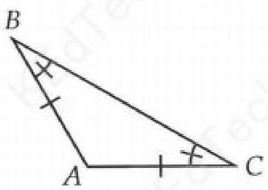
The area of a triangle is given by $\frac{1}{2}bh$, where b is the length of its base and h is its height. Any side of the triangle can be chosen as the base. Once the base is chosen, the height is defined as the perpendicular line segment from the opposite vertex (corner) of the triangle to the line containing the base. Here are some examples:



The altitude of a triangle is just another term for its height.

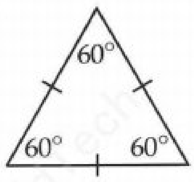
Isosceles & Equilateral Triangles

An isosceles triangle is one that has two sides of equal length. The angles opposite those sides are equal.



Because $AB = AC$, the measures of angles B and C are equal.

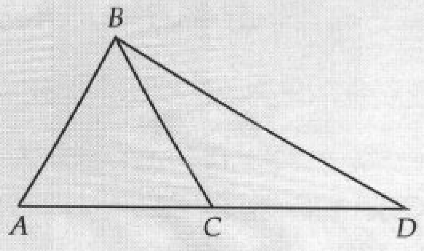
In an equilateral triangle, all sides have the same length. Because equal sides imply equal angles, each angle measures $180 \div 3 = 60°$.



EXAMPLE 1: In an isosceles triangle, the measure of one of the angles is 50°, and the measure of another angle is x°. What is the greatest possible value of x ?

An isosceles triangle has not only two equal sides but also two equal angles. There are two possibilities for an isosceles triangle with an angle of 50°: one of the unknown angles could be 50°, forming a 50–50–80 triangle, or the two unknown angles themselves could be equal, forming a 50–65–65 triangle. Given these two possibilities, $\boxed{80}$ is the greatest possible value of x.

EXAMPLE 2:

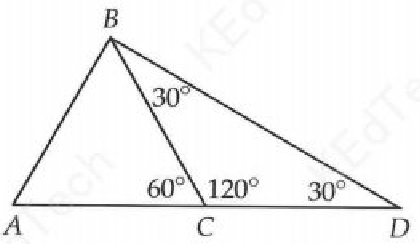


Note: Figure not drawn to scale.

In the figure, triangle ABC is equilateral and $BC = CD$. What is the measure, in degrees, of angle BDC ?

A) 15° B) 30° C) 45° D) 60°

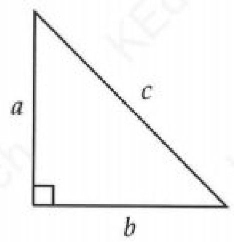
Each angle of an equilateral triangle measures 60°. Since angles ACB and BCD are supplementary (form a straight line), angle BCD measures $180 - 60 = 120^\circ$. Since $BC = CD$, angles CBD and BDC must have the same measure: $(180 - 120) \div 2 = 30^\circ$.



Answer $\boxed{(B)}$.

Right Triangles

Right triangles are made up of two legs and a hypotenuse (the side opposite the right angle). The hypotenuse is always the longest side.



The side lengths of any right triangle adhere to the **pythagorean theorem**: $a^2 + b^2 = c^2$, where a and b are the lengths of the legs and c is the length of the hypotenuse.

EXAMPLE 3: A right triangle has legs with lengths of 21 feet and 22 feet. What is the length, in feet, of the hypotenuse of this triangle?

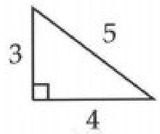
A) $5\sqrt{37}$ B) $15\sqrt{5}$ C) 43 D) 925

Let h be the length of the hypotenuse. Applying the pythagorean theorem, we get

$$21^2 + 22^2 = h^2$$
$$925 = h^2$$
$$h = \sqrt{925} = \sqrt{25 \cdot 37} = 5\sqrt{37}$$

Answer $\boxed{(A)}$.

When working with right triangles, certain side lengths come up repeatedly. For example, the 3–4–5 triangle:



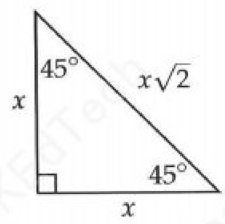
A set of three whole numbers that satisfy the pythagorean theorem is called a **pythagorean triple**. Though not necessary, it'll save you some time and effort if you learn to recognize the most common triples that show up:

3–4–5 · 6–8–10 · 5–12–13 · 7–24–25 · 8–15–17

Note that the 6–8–10 right triangle is just a multiple of the 3–4–5 right triangle.

Special Right Triangles

There are two special right triangles you have to know: the 45°–45°–90° triangle and the 30°–60°–90° triangle. They're "special" because of the way their sides relate to one another. Let's look at the 45–45–90 triangle first:



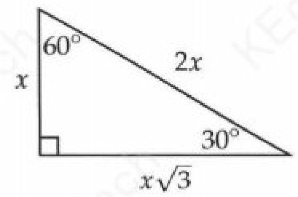
The 45–45–90 triangle is a right isosceles triangle, which means its two legs are equal. If the length of each leg is x, then the length of the hypotenuse is $x\sqrt{2}$ (square root 2 times longer than x).

Letting h be the length of the hypotenuse, we can prove this relationship using the pythagorean theorem:

$$x^2 + x^2 = h^2$$
$$2x^2 = h^2$$
$$\sqrt{2x^2} = \sqrt{h^2}$$
$$x\sqrt{2} = h$$

These proofs are shown not because they will be tested on the SAT, but because they illustrate important problem-solving concepts that you may have to use on certain SAT questions.

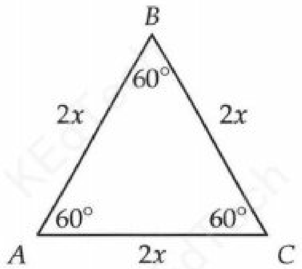
The second special right triangle is the $30°–60°–90°$ triangle:



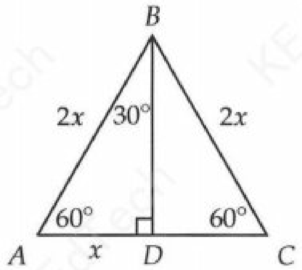
Because $30°$ is the smallest angle, the side opposite to it is the shortest. If we let the length of the shortest side be x, then the hypotenuse, the largest side, turns out to be twice as long as x, and the side opposite $60°$ turns out to be $\sqrt{3}$ times longer than x.

One common mistake students make is to think that because $60°$ is twice $30°$, the side opposite $60°$ must be twice as long as the side opposite $30°$. This line of thinking is flawed. You cannot extrapolate the ratio of the sides from the ratio of the angles. It's true that the side opposite $60°$ is longer than the side opposite $30°$, but it isn't twice as long.

We can prove the 30–60–90 relationship by using an equilateral triangle. Let each side be $2x$ (we could use x but you'll see why $2x$ makes things easier in a bit):



Drawing a perpendicular line from B to $\overline{AC}$ creates two 30–60–90 triangles:



Because an equilateral triangle is symmetrical, AD is half of $2x$, or just x. That's why we used $2x$—to avoid any fractions. We can now use the pythagorean theorem to find BD:

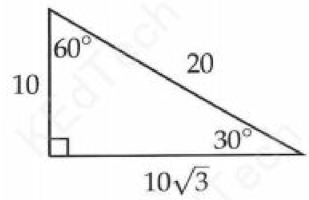
$$\begin{aligned} AD^2 + BD^2 &= AB^2 \\ x^2 + BD^2 &= (2x)^2 \\ BD^2 &= 4x^2 - x^2 \\ BD^2 &= 3x^2 \\ BD &= \sqrt{3x^2} = x\sqrt{3} \end{aligned}$$

The x, $x\sqrt{3}$, and $2x$ side lengths of triangle ABD are proof of the 30–60–90 triangle relationship.

EXAMPLE 4: In a right triangle, the measure of one of the acute angles is 30°, and the length of the hypotenuse is 20 inches. What is the area, in square inches, of this triangle?

A) $50\sqrt{3}$ B) 100 C) $100\sqrt{3}$ D) 200

Since the triangle has a right angle and a 30° angle, the remaining angle must be 60°. We can then use the 30–60–90 triangle relationship to get the following lengths:

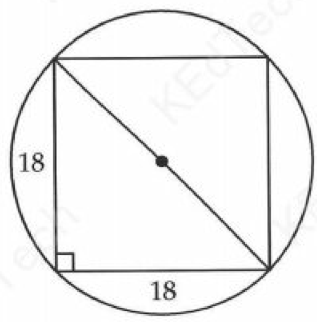


The area of this triangle is then $\frac{1}{2}bh = \frac{1}{2}(10\sqrt{3})(10) = 50\sqrt{3}$ square inches. Answer $\boxed{(A)}$.

EXAMPLE 5: A square with a side length of 18 centimeters is inscribed in a circle. What is the radius, in centimeters, of the circle?

A) 9 B) $9\sqrt{2}$ C) $12\sqrt{3}$ D) $18\sqrt{2}$

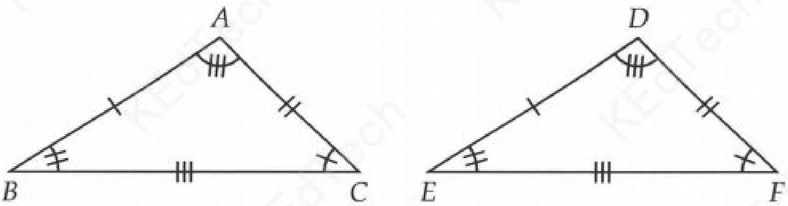
For any regular polygon inscribed in a circle, the center of the polygon is also the center of the circle. In this example, the center of the square is also the center of the circle. Because the centers are the same, a diagonal of the square is a diameter of the circle:



The diagonal splits the square into two 45–45–90 triangles (isosceles right triangles). According to the 45–45–90 triangle relationship, the length of the diagonal must be $18\sqrt{2}$ centimeters. The radius of the circle is then $18\sqrt{2} \div 2 = 9\sqrt{2}$ centimeters. Answer $\boxed{(B)}$.

Congruent Triangles

Two triangles are congruent if they have congruent corresponding angles and side lengths. For example, triangles ABC and DEF are congruent since $AB = DE$, $AC = DF$, $BC = EF$, $\angle A = \angle D$, $\angle B = \angle E$, and $\angle C = \angle F$.



Congruence implies that each vertex, angle, and side in one triangle has a matching counterpart in the other triangle. In math, we call these matching pairs *corresponding vertices*, *corresponding sides*, and *corresponding angles*. For instance, vertex A *corresponds to* vertex D, $\angle A$ *corresponds to* $\angle D$, side $\overline{AB}$ *corresponds to* side $\overline{DE}$, and so on.

Congruent triangles are named so that corresponding vertices/angles are in the same order. Take a look at how the congruent triangles ABC and DEF are named. The first letters are the corresponding vertices A and D. The second letters are the corresponding vertices B and E. Finally, the last letters are the corresponding vertices C and F. Mathematically, $\triangle ABC \cong \triangle DEF$. It would not be correct to write $\triangle ABC \cong \triangle EDF$ since A and E do not correspond and B and D do not correspond.

Because congruent triangles are named this way, we can determine which sides correspond to which just by looking at the names. For triangles ABC and DEF, the first two letters are AB and DE, respectively, so $\overline{AB}$ corresponds to $\overline{DE}$. Similarly, the last two letters are BC and EF, respectively, so $\overline{BC}$ corresponds to $\overline{EF}$. Finally, the first and last letters are AC and DF, respectively, so $\overline{AC}$ corresponds to $\overline{DF}$.

EXAMPLE 6: Triangles ABC and DEF are congruent, where A corresponds to D, and angles B and E each have measure 50°. The measure of angle C is 25°. What is the measure of angle D ?

A) 25° B) 65° C) 75° D) 105°

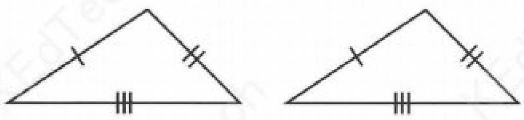
Since congruent triangles have congruent corresponding angles and angle C (25°) corresponds to angle F, the measure of angle F must also be 25°. The measure of angle D is then $180^\circ - \angle E - \angle F = 180 - 50 - 25 = 105^\circ$. Answer $\boxed{(D)}$.

Triangle Congruence Theorems

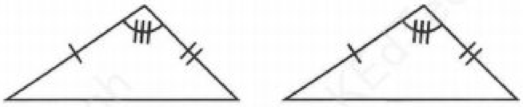
Now how do we prove that two triangles are congruent? The process would be very tedious if we had to check every side and every angle. Fortunately, there are ways of proving congruence without knowing all the sides and angles. Once two triangles are proven to be congruent, then all their corresponding parts are congruent.

Triangle congruence theorems express the minimum amount of information needed to prove that two triangles are congruent:

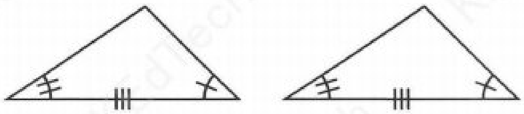
- SSS (side-side-side): If all pairs of corresponding sides are congruent, then the triangles are congruent.



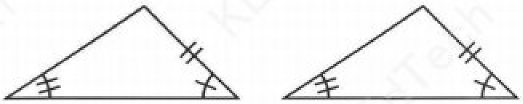
- SAS (side-angle-side): If two pairs of corresponding sides are congruent and the corresponding angles between those sides are congruent, then the triangles are congruent.



- ASA (angle-side-angle): If two pairs of corresponding angles are congruent and the corresponding sides between those angles are congruent, then the triangles are congruent.

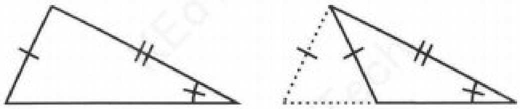


- AAS (angle-angle-side): If two pairs of corresponding angles are congruent and the corresponding sides opposite to one of those pairs of angles are congruent, then the triangles are congruent.



ASA and AAS are essentially the same because if you know two angles of a triangle, you know the third. Put simply, two angles and a side are enough to prove congruence.

Note that SSA (side-side-angle) is NOT a triangle congruence theorem because it only proves congruence when the angle is at least 90°. When the angle is acute, more than one triangle is possible, so congruence isn't guaranteed. For example, the following two triangles satisfy SSA but are obviously not congruent.

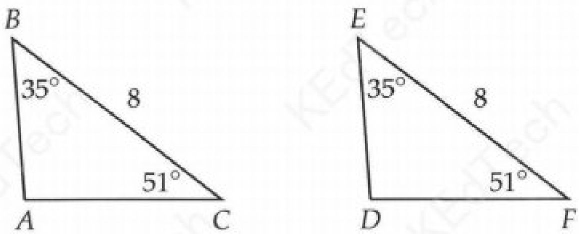


You will not be tested on the nuances of SSA. Just know that it is not a triangle congruence theorem.

EXAMPLE 7: In triangles ABC and DEF, angles C and F each have measure $51°$, $BC = 8$, and $EF = 8$. Which additional piece of information is sufficient to prove that triangle ABC is congruent to triangle DEF?

A) $AB = 9$ and $DE = 9$
B) $AC = 14$ and $DE = 14$
C) Angles B and E each have measure 35°.
D) Angles B and D each have measure 43°.

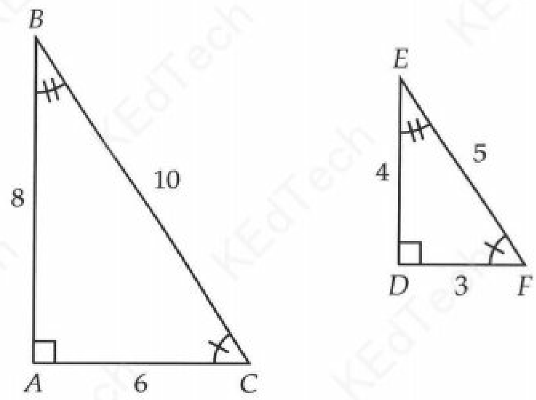
Angles B and E are corresponding angles. If they had the same measure, then the two triangles would be congruent by ASA (angle-side-angle).



Answer $\boxed{\text{(C)}}$. Answer A is wrong because SSA is not a valid triangle congruence theorem. Answer B is wrong because $\overline{AC}$ and $\overline{DE}$ are not corresponding sides—their congruence doesn't help satisfy any of the triangle congruence theorems. Answer D is wrong because the given measures do not result in congruent corresponding angles between the two triangles.

Similar Triangles

Two triangles are similar when their corresponding angles are congruent and their corresponding sides are proportional in the same ratio. Put simply, similar triangles are different size versions of one another.



In similar triangles ABC and DEF above, $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$, $AB = 2DE$, $AC = 2DF$, and $BC = 2EF$. As a result, triangle ABC is a larger version of triangle DEF—the triangles have the same shape but not the same size.

With similar triangles, we use the same naming scheme as we do with congruent triangles. If $\triangle ABC \sim \triangle DEF$ (the $\sim$ symbol means "similar"), then A and D, the first respective letters, are corresponding vertices, B and E, the second respective letters, are corresponding vertices, and C and F, the last respective letters, are corresponding vertices. The same logic applies when determining which sides correspond to which. The first two respective letters of each name, AB and DE, indicate that side $\overline{AB}$ corresponds to side $\overline{DE}$, and so on.

It would not be correct to write $\triangle ABC \sim \triangle EDF$ since A and E do not correspond and B and D do not correspond.

In summary, the two most important aspects of similar triangles is that 1) their corresponding angles are congruent and 2) their corresponding sides are proportional in the same ratio. In our example above,

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = 2$$

This equation also implies that

$$\frac{AB}{AC} = \frac{DE}{DF} \quad \text{and} \quad \frac{AB}{BC} = \frac{DE}{EF} \quad \text{and} \quad \frac{AC}{BC} = \frac{DF}{EF}$$

In other words, not only are ratios between corresponding sides equivalent but "corresponding ratios" between sides are also equivalent.

Setting up these types of equations will be your goal in many of the harder similar triangles questions.

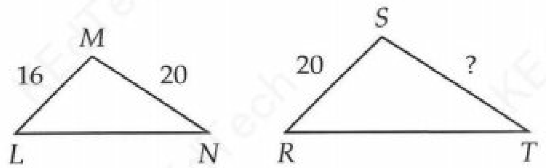
EXAMPLE 8: Isosceles triangles FGH and JKL are similar, where F and G correspond to J and K, respectively. Angle G has a measure of $68°$. What is the measure of angle K ?

A) $44°$ B) $56°$ C) $68°$ D) $112°$

Since similar triangles have congruent corresponding angles and angle G ($68°$) corresponds to angle K, the measure of angle K must also be $68°$. Answer $\boxed{\text{(C)}}$. The fact that the triangles are isosceles is irrelevant.

EXAMPLE 9: Triangle LMN is similar to RST such that L, M, and N correspond to R, S, and T, respectively. The length of $\overline{LM}$ is 16 and the lengths of $\overline{MN}$ and $\overline{RS}$ are each 20. What is the length of $\overline{ST}$?

A) 16 B) 20 C) 24 D) 25



Similar triangles have corresponding sides in the same proportion. Since $\overline{RS}$ and $\overline{LM}$ are corresponding sides, triangle RST must have sides that are $\frac{20}{16} = \frac{5}{4}$ times longer than the corresponding sides of triangle LMN. Therefore, the length of $\overline{ST}$ is $\frac{5}{4}$ times longer than corresponding side $\overline{MN}$: $ST = \frac{5}{4}MN = \frac{5}{4}(20) = 25$.

Answer $\boxed{(D)}$. Here is the solution expressed as an equation that can be solved for ST:

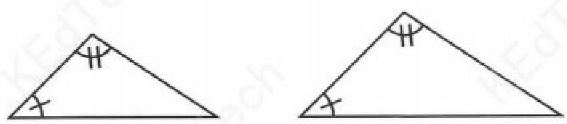
$$\frac{RS}{LM} = \frac{ST}{MN} \quad \Rightarrow \quad \frac{20}{16} = \frac{ST}{20} \quad \Rightarrow \quad ST = \frac{20}{16}(20) = 25$$

Triangle Similarity Theorems

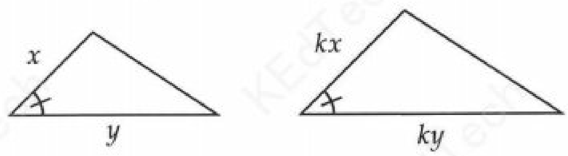
Now how do we determine whether two triangles are similar in the first place? Just as we have triangle congruence theorems, we have triangle similarity theorems to help us prove similarity without needing to check every side and angle. Once two triangles are proven to be similar, then all corresponding angles are congruent and all corresponding sides are proportional according to the same ratio.

Triangle similarity theorems express the minimum amount of information needed to prove that two triangles are similar:

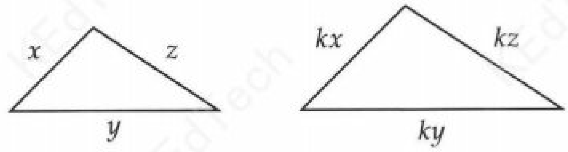
- AA(A) (angle-angle-(angle)): If two pairs of corresponding angles are congruent, then the triangles are similar. Note that AA is the same as AAA because if two pairs of angles are congruent, then the third pair must also be congruent.



- SAS (side-angle-side): If two pairs of corresponding sides are proportional according to the same ratio and the corresponding angles between those sides are congruent, then the triangles are similar.



- SSS (side-side-side): If all pairs of corresponding sides are proportional according to the same ratio, then the triangles are similar.



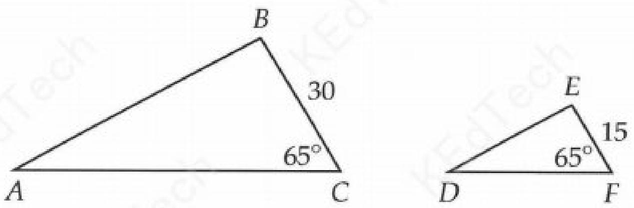
Among the three similarity theorems, AA (angle-angle) is by far the most commonly tested.

EXAMPLE 10: In triangle ABC, the measure of angle C is 65° and the length of side BC is 30. In triangle DEF, the measure of angle F is 65° and the length of side EF is 15. Which additional piece of information is sufficient to prove that triangle ABC is similar to triangle DEF ?

I. The length of side AB is twice the length of side DE.
II. The length of side AC is twice the length of side DF.

A) I is sufficient, but II is not.
B) II is sufficient, but I is not.
C) Either I or II is sufficient.
D) Neither I nor II is sufficient.

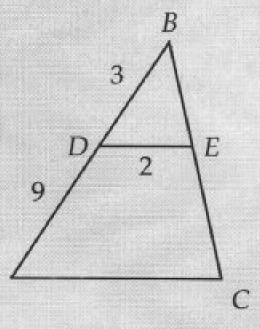
Let's start with a picture, which isn't necessarily drawn to scale.



If statement I ($AB = 2DE$) were true, then we would have two pairs of corresponding sides in the same proportion. However, SSA (side-side-angle) is not a valid triangle similarity theorem, so I is not sufficient.

Now if statement II ($AC = 2DF$) were true, then we would again have two pairs of corresponding sides in the same proportion. This time, however, the congruent angles would lie between the sides in each triangle. Since SAS (side-angle-side) is a valid triangle similarity theorem, II is sufficient. Answer $\boxed{(B)}$.

EXAMPLE 11:



Note: Figure not drawn to scale.

In triangle ABC, $\overline{DE}$ is parallel to $\overline{AC}$, $AD = 9$, $DB = 3$, and $DE = 2$. What is the length of $\overline{AC}$?

Because $\overline{DE}$ and $\overline{AC}$ are parallel, $\angle BDE$ is congruent to $\angle BAC$ and $\angle BED$ is congruent to $\angle BCA$. Therefore, triangle BDE and triangle BAC are similar by AA (angle-angle).

Now the answer is NOT 6. A common mistake that students make whenever they see similar triangles is to assume that certain segments correspond to each other when they do not. Of course, some questions are designed so that this mistake is easy to make. In this particular case, it's easy to assume that because AD is 3 times BD, AC must be 3 times DE, but this would be incorrect. When figuring out the sides of similar triangles, we have to look at, well, the actual *sides* of the triangles, and $\overline{AD}$ is not a side of any triangle, just a portion of one.

One correct approach is to set up an equation that relates the relevant corresponding sides:

$$\frac{BD}{BA} = \frac{DE}{AC}$$
$$\frac{3}{3+9} = \frac{2}{AC}$$

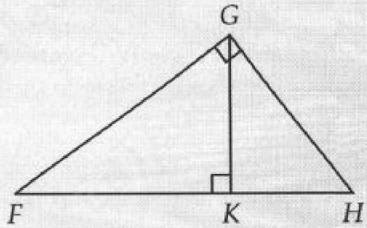
Cross multiplying, we get

$$3AC = 24$$
$$AC = \boxed{8}$$

In this example, the similarity between the two triangles was easy to recognize, and it was obvious which sides corresponded to each other. In tougher questions, similar triangles and their corresponding sides are more difficult to spot and keep track of.

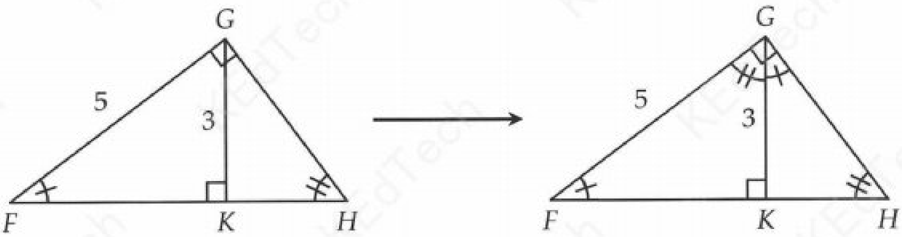
For these tougher questions, I recommend labeling congruent angles with tick marks. The tick marks will help you determine which vertices correspond and which sides correspond. Here's an example to illustrate:

EXAMPLE 12:



In the figure, $FG = 5$ and $GK = 3$. What is the length of $\overline{KH}$?

At first glance, this does not look like a similar triangles question. But if we label the angles of the outside triangle FGH, something interesting happens. Let's first label $\angle F$ with one tick mark and $\angle H$ with two tick marks.



Now whenever two angles of one triangle are equal in measure to two angles of another, the remaining angles must also be equal to each other. Because outside triangle FGH and triangle FGK on the left both have right angles and share $\angle F$, $\angle FGK$ must have the same measure as $\angle H$ (two tick marks). Likewise, outside triangle FGH and triangle GKH on the right both have right angles and share $\angle H$, so $\angle KGH$ must have the same measure as $\angle F$ (one tick mark).

The result is that outside triangle FGH, triangle FKG on the left, and triangle GKH on the right all have congruent angles, which means they're all similar to one another by AA(A) (angle-angle-(angle))!

Note that the tick marks helped us name these triangles by making it easy to identify the corresponding vertices. If the outer triangle is named FGH (1-tick, right angle, 2-tick), then the triangle on the left should be named FKG (1-tick, right angle, 2-tick).

Since triangle FKG is a 3–4–5 triangle with $FK = 4$, an equation that relates the corresponding sides of triangles FKG and GKH would allow us to solve for KH.

To identify the corresponding sides, we can use the order of the letters in the triangles' names, as discussed earlier, or we can use our tick marks: sides opposite from angles with the same number of tick marks correspond to each other.

$$\frac{KH \text{ (opposite 1-tick angle in } \triangle GKH)}{KG \text{ (opposite 1-tick angle in } \triangle FKG)} = \frac{GK \text{ (opposite 2-tick angle in } \triangle GKH)}{FK \text{ (opposite 2-tick angle in } \triangle FKG)}$$

$$\frac{KH}{3} = \frac{3}{4}$$

$$KH = \boxed{\frac{9}{4}}$$

Similarity in Other Shapes

Triangles aren't the only shapes that can be similar to one another. Since similarity exists when there are different size versions of the same shape, other polygons can exhibit similarity as well.

When dealing with similar polygons, just remember these fundamental properties of similarity:

- Congruent corresponding angles
- Corresponding sides that are proportional in the same ratio

EXAMPLE 13: Quadrilateral $ABCD$ is similar to quadrilateral $WXYZ$, where A, B, C and D correspond to W, X, Y, and Z, respectively. The measure of angle A is $50°$, the measure of angle B is $100°$, and the measure of angle Y is $140°$. What is the measure of angle Z ?

A) $50°$ B) $70°$ C) $90°$ D) $120°$

Since corresponding angles are congruent and angles A ($50°$) and B ($100°$) correspond to angles W and X, respectively, the measure of angle W must be $50°$ and the measure of angle X must be $100°$. The measure of angle Z is then $360° - \angle W - \angle X - \angle Y = 360 - 50 - 100 - 140 = 70°$. Answer $\boxed{(B)}$.

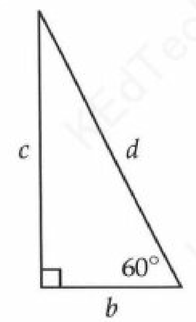
EXERCISE 1: Spare paper recommended. Answers for this exercise start on page 381.

1

A square has a side length of 6 inches. What is the length, in inches, of the square's diagonal?

A) $3\sqrt{2}$
B) 6
C) $6\sqrt{2}$
D) $6\sqrt{3}$

2



Note: Figure not drawn to scale.

For the right triangle shown, $d = 10$. What is the value of b ?

A) 5
B) $5\sqrt{2}$
C) $5\sqrt{3}$
D) $10\sqrt{3}$

3

Triangles LMN and PQR are congruent, where L corresponds to P and M corresponds to Q. The measures of angles L and R are 94° and 22°, respectively. What is the measure of angle M ?

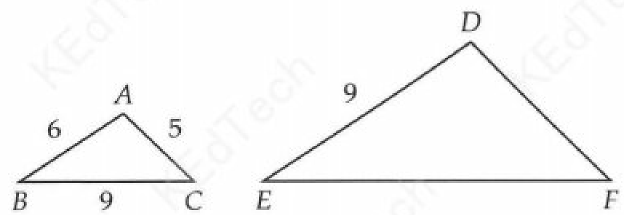
A) 22°
B) 64°
C) 94°
D) 116°

4

Triangles ABC and KLM are similar, where A and B correspond to K and L, respectively. The measure of angle A is 15°, $BC = 24$, and $KM = 3AC$. What is the length of $\overline{LM}$?

A) 8
B) 45
C) 72
D) 216

5



In the figure shown, triangle ABC is similar to triangle DEF. What is the perimeter of triangle DEF ?

A) 20
B) 26.8
C) 30
D) 36.2

6

Triangles ABC and JKL are similar, where A corresponds to J, and B and K are right angles. The measure of angle A is 34°, and $KL = \frac{1}{2}BC$. What is the measure of angle L ?

A) 17
B) 28
C) 34
D) 56

7

A right triangle has a hypotenuse of length 31 inches. One of the legs of this triangle has a length of 25 inches and the other leg has a length of $4\sqrt{n}$ inches, where n is an integer constant. What is the value of n ?

8

A right triangle has a hypotenuse of length 42 centimeters and an angle measuring 30°. What is the perimeter, in centimeters, of this triangle?

A) $21 + 21\sqrt{3}$
B) $56 + 14\sqrt{3}$
C) $63 + 21\sqrt{3}$
D) $63 + 42\sqrt{3}$

9

In triangles LMN and RST, angles L and R each have measure 127°. Which additional piece of information is sufficient to determine whether triangle LMN is similar to triangle RST ?

A) The measures of angles N and T
B) The lengths of sides LM and RS
C) The measures of angles S and T
D) The lengths of sides MN and ST

10

When adjusted to a height of 35 feet, a pole casts a shadow that is 14 feet long. By how many feet must the height of the pole be increased so that it casts a shadow that is 18 feet long?

11

In triangle XYZ, the measure of angle Y is 70°. Which of the following pieces of information is sufficient to prove that triangle XYZ is an isosceles triangle?

I. The measure of angle X is 40°.
II. The measure of angle Z is 55°.

A) I is sufficient, but II is not.
B) II is sufficient, but I is not.
C) I is sufficient and II is sufficient.
D) Neither I nor II is sufficient.

12

The hypotenuse of an isosceles right triangle has a length of 12 units. What is the area of the triangle, in square units?

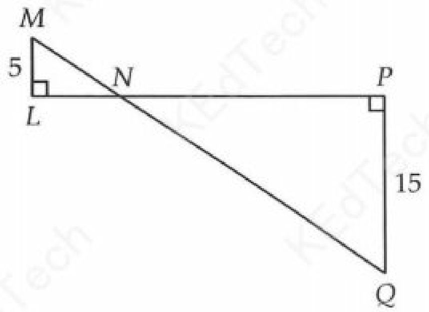
A) $18\sqrt{2}$
B) 36
C) 48
D) $36\sqrt{2}$

13

In triangles ABC and DEF, angles A and D each have measure 72° and angles B and E each have measure 36°. Which additional piece of information is sufficient to prove that triangle ABC is congruent to triangle DEF ?

A) The measure of angle A is equal to the measure of angle C.
B) The length of side AC is equal to the length of side DF.
C) The measure of angle C is equal to the measure of angle F.
D) The length of side DE is equal to the length of side EF.

14



In the figure, $\overline{MQ}$ and $\overline{LP}$ intersect at point N, and $MQ = 38$. What is the length of $\overline{NQ}$?

15

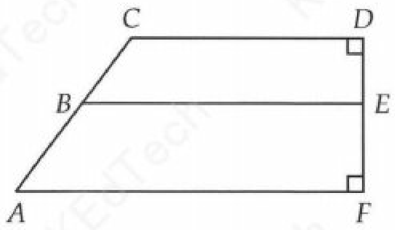
In triangles CDE and FGH, angles C and F each have measure $75°$, $CD = 20$, and $FG = 5$. Which additional piece of information is sufficient to prove that triangle CDE is similar to triangle FGH ?

A) The measures of angles D and H are $64°$ and $16°$, respectively.
B) The measures of angles E and G are $70°$ and $35°$, respectively.
C) $CE = 8$ and $FH = 4$
D) $DE = 16$ and $GH = 4$

16

An equilateral triangle has a perimeter of $42\sqrt{3}$ inches. What is the height, in inches, of this triangle?

17



Note: Figure not drawn to scale.

In the figure, trapezoid $BCDE$ is similar to trapezoid $ABEF$. The length of $\overline{AB}$ is 4, the length of $\overline{DE}$ is 2, and the length of $\overline{BC}$ is equal to the length of $\overline{EF}$. What is the length of $\overline{EF}$?

A) $\sqrt{6}$
B) $2\sqrt{2}$
C) $2\sqrt{3}$
D) 3

18

A right triangle has an area of 450 square inches. The triangle has sides of length $a\sqrt{3}$, $a\sqrt{12}$, and $3a$ inches, where a is a constant. To the nearest tenth of an inch, what is the length of the shortest side of this triangle?

A) 16.1
B) 19.4
C) 21.2
D) 22.8

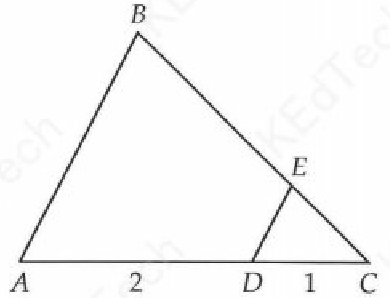
19

In triangle DEF, the measure of angle D is $90°$ and $\overline{DG}$ is an altitude of the triangle. The length of $\overline{DE}$ is 5, and the length of $\overline{EG}$ is 4. Which of the following ratios has a value of $\frac{3}{4}$?

A) $\frac{DF}{EF}$
B) $\frac{DG}{DF}$
C) $\frac{DG}{FG}$
D) $\frac{FG}{DG}$

EXERCISE 2: Spare paper recommended. Answers for this exercise start on page 383.

1

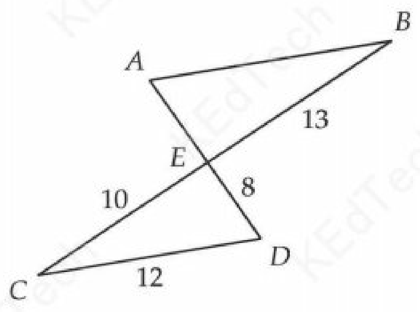


Note: Figure not drawn to scale.

In the figure shown, triangle ABC is similar to triangle DEC such that A corresponds to D and B corresponds to E. The measure of $\angle ABC$ is 120°. What is the measure of $\angle DEC$?

A) 40°
B) 60°
C) 80°
D) 120°

2



Note: Figure not drawn to scale.

In the figure shown, $\overline{AD}$ and $\overline{BC}$ intersect at point E, and $\overline{AB}$ is parallel to $\overline{CD}$. What is the length of $\overline{AB}$?

A) $\frac{130}{8}$
B) $\frac{130}{12}$
C) $\frac{156}{8}$
D) $\frac{156}{10}$

3

Triangles ABC and DEF are similar, where A corresponds to D and B corresponds to E. The measure of angle A is 20°, $EF = 6$, and $DF = 6$. What is the measure of angle C ?

A) 20°
B) 80°
C) 140°
D) 160°

4

In triangles RST and WXY, angles R, T, W, and Y each have measure 29°. Which additional piece of information is sufficient to prove that triangle RST is congruent to triangle WXY ?

A) The measures of angle S and angle X are equal.
B) The lengths of sides RT and WX are equal.
C) The lengths of sides RT and WY are equal.
D) No additional information is necessary.

5

A right triangle has legs with lengths of $16\sqrt{23}$ inches and $30\sqrt{23}$ inches. What is the length of the hypotenuse of this triangle, in inches?

A) $34\sqrt{23}$
B) $36\sqrt{23}$
C) $38\sqrt{23}$
D) $42\sqrt{23}$

6

Right triangles ABC and DEF are similar, where A and D are right angles, and B corresponds to E. If $AB = 12$, $BC = 60\sqrt{2}$, and $DF = 105$, what is the length of side EF ?

A) $48\sqrt{2}$
B) $75\sqrt{2}$
C) $100\sqrt{2}$
D) $525\sqrt{2}$

7

Isosceles triangles ABC and DEF are similar, where A corresponds to D and B corresponds to E. The measure of angle A is equal to the measure of angle B, $AB = 10$, $AC = 5$, and $EF = 40$. What is the length of side DE ?

A) 20
B) 40
C) 60
D) 80

8

$$A(x) = \frac{1}{2}x(7x)$$

The function A gives the area, in square feet, of a certain right triangle, where x is the length, in feet, of one of the legs of the triangle. If the length of this triangle's hypotenuse, in feet, can be written in the form $ax\sqrt{b}$, where a and b are integers, what is the value of $a + b$?

9

In triangle ABC, the measure of angle A is $30°$ and $AB = AC$. In triangle DEF, the measure of angle E is $75°$. Which additional piece of information is sufficient to determine whether triangle ABC is similar to triangle DEF ?

A) The lengths of $\overline{AB}$ and $\overline{DE}$
B) The lengths of $\overline{BC}$ and $\overline{EF}$
C) The measure of angle B
D) The measure of angle F

10

According to the triangle inequality theorem, the length of any side of a triangle must be smaller than the sum of the lengths of the other two sides. If a triangle has side lengths of 9 inches, 15 inches, and x inches, which inequality represents all possible values of x ?

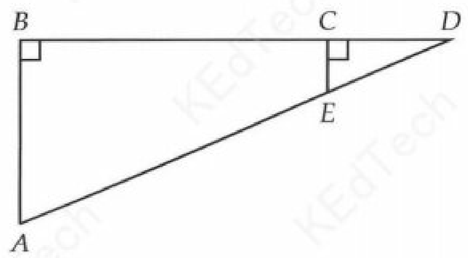
A) $0 < x < 24$
B) $6 < x < 24$
C) $12 < x < 24$
D) $x > 24$

11

In triangles LMN and PQR, angles M and Q each have measure $54°$, $MN = 16$, and $QR = 16$. Which additional piece of information is sufficient to prove that triangle LMN is congruent to triangle PQR ?

A) Angles L and R each have measure $44°$.
B) Angles N and P each have measure $44°$.
C) $LM = 11$ and $PQ = 11$
D) $LN = 11$ and $PR = 11$

12



Note: Figure not drawn to scale.

In the figure shown, $CE = 9$ and $BC = 6CD$. What is the length of line segment AB ?

13

A circle is inscribed in an equilateral triangle. The radius of the circle is 9 inches. What is the perimeter, in inches, of the triangle?

A) $18\sqrt{3}$
B) $27\sqrt{3}$
C) $54\sqrt{3}$
D) 108

14

In triangle ABC, angle B measures $90°$, $AC = 106$, and $BC = 90$. Point D on side AC is connected by a line segment with point E on side BC such that line segment DE is parallel to side AB, and $DE = 14$. What is the length of line segment BE ?

15

In triangle FGH, $FG = 21$ and $GH = 27$. In triangle XYZ, $XY = 28$ and $YZ = 36$. Which additional piece of information is sufficient to prove that triangles FGH and XYZ are similar?

I. The measure of angle G is equal to the measure of angle Y.
II. $FH = 15$ and $XZ = 20$

A) I is sufficient, but II is not.
B) II is sufficient, but I is not.
C) I is sufficient and II is sufficient.
D) Neither I nor II is sufficient.

16

An equilateral triangle is inscribed in a circle. The triangle has a side length of $22\sqrt{3}$ feet. What is the radius, in feet, of the circle?

A) $11\sqrt{3}$
B) 22
C) 33
D) 44

17

In triangle ABC, angle A is a right angle, point D lies on $\overline{AC}$, point E lies on $\overline{BC}$, and $\overline{DE}$ is parallel to $\overline{AB}$. The length of $\overline{DE}$ is 36 inches, the length of $\overline{AB}$ is 54 inches, and the length of $\overline{AD}$ is 15 inches. What is the area, in square inches, of triangle CDE ?

18

In triangle ABC, angle B is a right angle, point D lies on $\overline{AB}$, point E lies on $\overline{AC}$, and $\overline{DE}$ is perpendicular to $\overline{AC}$. If the length of $\overline{AB}$ is 12 units, the length of $\overline{AC}$ is 13 units, and the length of $\overline{DE}$ is 3 units, what is the length of $\overline{AE}$, in units?

19

In triangle ABC, $AB = 6$, $BC = 6$, and the measure of angle ABC is $120°$. What is the area of triangle ABC ?

A) $2\sqrt{3}$
B) $4\sqrt{3}$
C) $6\sqrt{3}$
D) $9\sqrt{3}$

20

Square $ABCD$ has a side length of 12. Point F lies on $\overline{BC}$, and $\overline{AF}$ and $\overline{BD}$ intersect at point E. The length of $\overline{BF}$ is 4. What is the length of $\overline{BE}$?

A) 3
B) $2\sqrt{2}$
C) $3\sqrt{2}$
D) $4\sqrt{2}$

21

A right triangle has a perimeter of $78 + 78\sqrt{3}$ centimeters and an angle measuring $30°$. What is the length, in centimeters, of the shortest side of this triangle?

A) 26
B) $26\sqrt{3}$
C) 39
D) $39\sqrt{3}$

22

In triangle QRS, angle Q is a right angle and $\overline{QT}$ is an altitude of the triangle. The length of $\overline{QS}$ is 15 units and the length of $\overline{RS}$ is 17 units. What is the length of $\overline{TS}$ to the nearest tenth of a unit?

A) 12.6
B) 12.8
C) 13.2
D) 13.4

23

In triangle WXY, point Z lies on $\overline{WY}$, and the measures of $\angle W$ and $\angle YXZ$ are each $36°$. Which of the following ratios is equal to $\frac{WY}{XY}$?

A) $\frac{WX}{XZ}$
B) $\frac{XY}{WX}$
C) $\frac{XY}{YZ}$
D) $\frac{YZ}{XZ}$