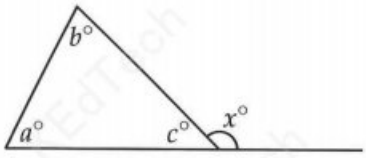
17 Angles

Exterior Angle Theorem

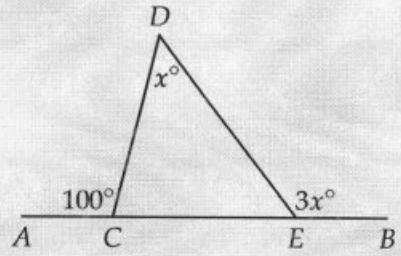
An exterior angle is formed when any side of a triangle is extended. In the triangle below, x° designates an exterior angle.



An exterior angle is always equal to the sum of the two angles in the triangle furthest from it:

$$x = a + b$$

EXAMPLE 1:

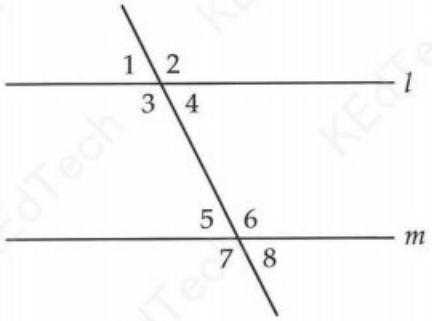


In the figure, points C and E lie on $\overline{AB}$. What is the value of x ?

Since $\angle ACD$ and $\angle DCE$ lie on the same side of a straight line, they are supplementary (sum to 180°). The measure of $\angle DCE$ is then $180 - 100 = 80°$. Now, $\angle DEB$ is an exterior angle to $\angle DCE$ and $\angle CDE$. Using the exterior angle theorem, we get

$$80 + x = 3x$$
$$80 = 2x$$
$$x = \boxed{40}$$

Parallel Lines



When a line intersects two parallel lines, the following are true:

- Vertical angles are congruent (e.g. $\angle 1 \cong \angle 4$ and $\angle 6 \cong \angle 7$).
- Alternate interior angles are congruent (e.g. $\angle 3 \cong \angle 6$ and $\angle 4 \cong \angle 5$).
- Alternate exterior angles are congruent (e.g. $\angle 1 \cong \angle 8$ and $\angle 2 \cong \angle 7$).
- Corresponding angles are congruent (e.g. $\angle 1 \cong \angle 5$ and $\angle 4 \cong \angle 8$).
- Same side interior angles are supplementary (e.g. $\angle 3 + \angle 5 = 180^\circ$ and $\angle 4 + \angle 6 = 180^\circ$).

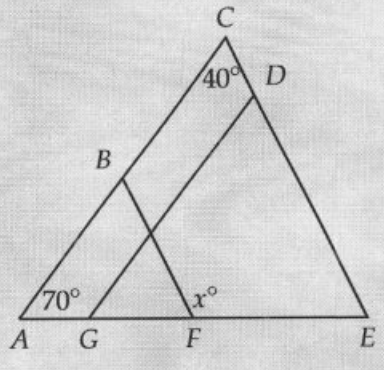
Essentially, when a line crosses two parallel lines, there are two sets of congruent angles:

$$\angle 1 \cong \angle 4 \cong \angle 5 \cong \angle 8$$
$$\angle 2 \cong \angle 3 \cong \angle 6 \cong \angle 7$$

Given two lines intersected by a third line, the two lines must be parallel if any one of the following congruencies exist:

- Congruent alternate interior angles (e.g. $\angle 4 \cong \angle 5$)
- Congruent alternate exterior angles (e.g. $\angle 2 \cong \angle 7$)
- Congruent corresponding angles (e.g. $\angle 2 \cong \angle 6$)

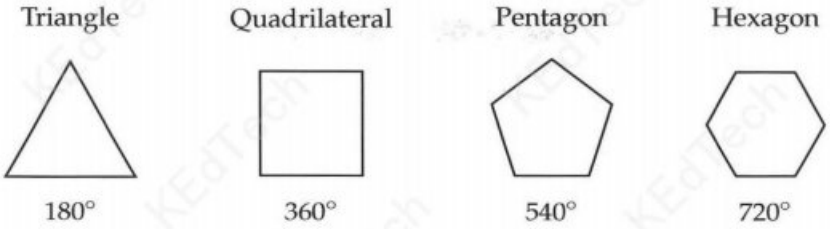
EXAMPLE 2:



In the figure, $\overline{AC}$ is parallel to $\overline{GD}$, and $\overline{BF}$ is parallel to $\overline{CE}$. What is the value of x ?

Here is the fastest approach: $\angle ACE = \angle ABF = 40^\circ$ because they are corresponding angles ($\overline{AC}$ cuts parallel lines $\overline{BF}$ and $\overline{CE}$). Since $\angle BFE$ is an exterior angle to $\triangle ABF$, $x = 70 + 40 = \boxed{110}$ by the exterior angle theorem.

Polygons



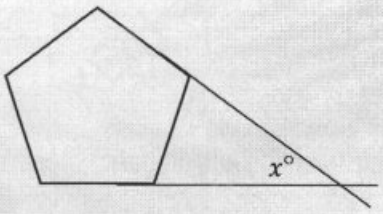
As you can see, each additional side in a polygon increases the sum of the interior angles by 180°. For any polygon, the sum of the interior angles is given by

$$180(n-2) \text{ where } n \text{ is the number of sides}$$

A **regular polygon** is a polygon in which all sides and angles are equal. The polygons shown above are regular. To find the measure of each interior angle in a regular polygon, divide the sum of the interior angles by the number of interior angles. A regular hexagon, for example, has interior angles that each measure $720 \div 6 = 120°$.

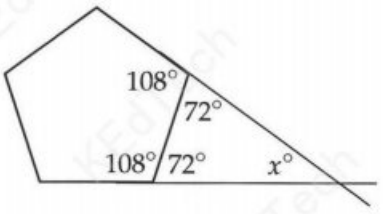
On the SAT, you are mainly tested on triangles and quadrilaterals, but it's good to know the basics for any polygon.

EXAMPLE 3:



In the figure, two sides of a regular pentagon are extended to form a triangle. What is the value of x ?

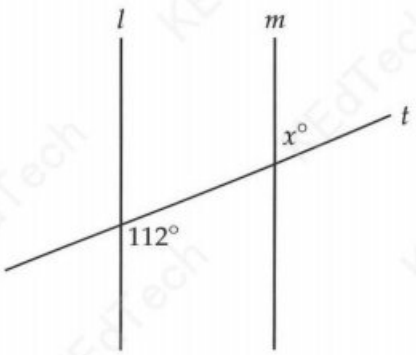
The total number of degrees in a pentagon is $180(5-2) = 540°$, so each interior angle measures $540° \div 5 = 108°$. The supplementary angles are then $180 - 108 = 72°$.



Since the angles in a triangle sum to 180°, $x = 180 - 72 - 72 = \boxed{36}$.

EXERCISE: Answers for this exercise start on page 379.

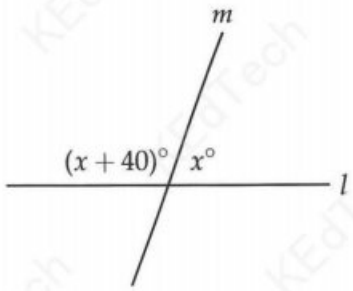
1



Note: Figure not drawn to scale.

In the figure, line l is parallel to line m. What is the value of x ?

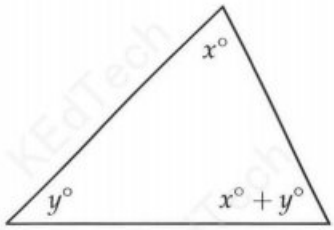
2



In the figure, line l intersects line m. What is the value of x ?

A) 60
B) 70
C) 75
D) 80

3

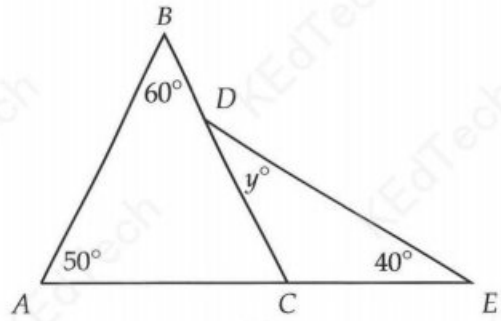


Note: Figure not drawn to scale.

In the figure, $x = 40$. What is the value of y ?

A) 40
B) 50
C) 80
D) 90

4

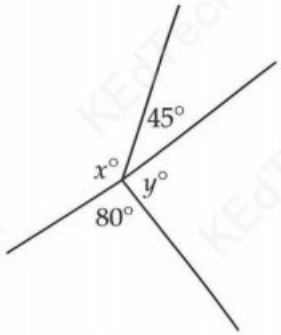


Note: Figure not drawn to scale.

In the figure, point C lies on $\overline{AE}$, and point D lies on $\overline{BC}$. What is the value of y ?

A) 30
B) 40
C) 50
D) 70

5

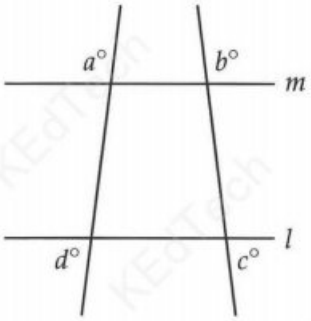


Note: Figure not drawn to scale.

In the figure, four line segments intersect at a point. What is the value of $x + y$?

A) 125
B) 180
C) 235
D) 280

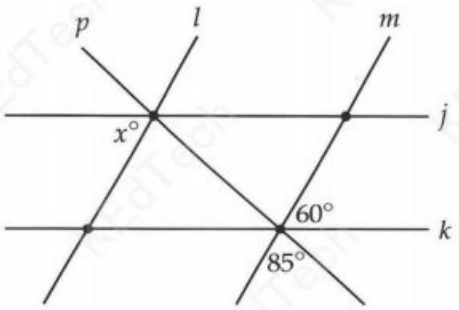
6



In the figure, lines l and m are parallel. What is the value of $a + b + c + d$?

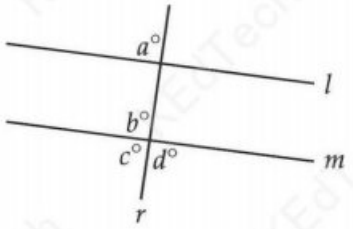
A) 270
B) 360
C) 720
D) It cannot be determined from the information given.

7



In the figure, lines j and k are parallel, and lines l and m are parallel. What is the value of x ?

8

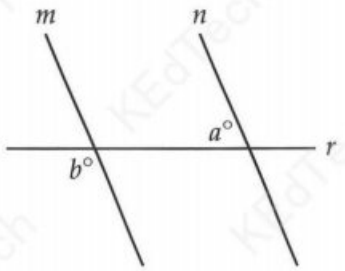


Note: Figure not drawn to scale.

In the figure, line r intersects lines l and m. Which of the following is sufficient to prove that lines l and m are parallel?

A) $a = c$
B) $a = d$
C) $b = c$
D) $b = d$

9

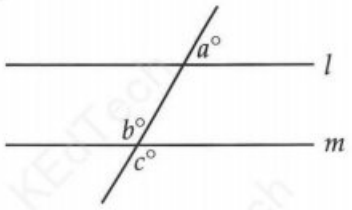


Note: Figure not drawn to scale.

In the figure shown, lines m and n are parallel, and line r intersects both lines. If $a > 74$, which of the following must be true?

A) $b < 106$
B) $b > 106$
C) $b < 180 - a$
D) $b > 180 - a$

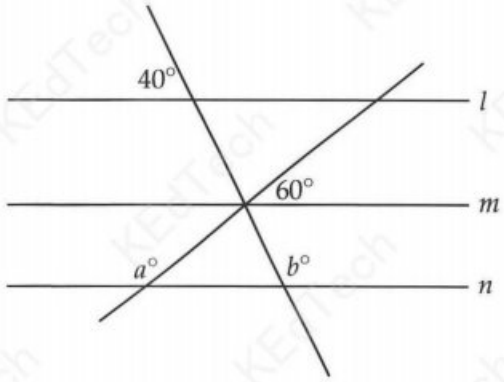
10



Note: Figure not drawn to scale.

In the figure, lines l and m are parallel. If $b = 9k + 15$ and $c = 16k - 76$, what is the value of a ?

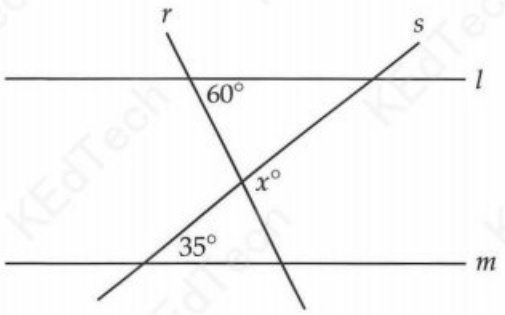
11



Note: Figure not drawn to scale.

In the figure, lines l, m, and n are parallel. What is the value of $a + b$?

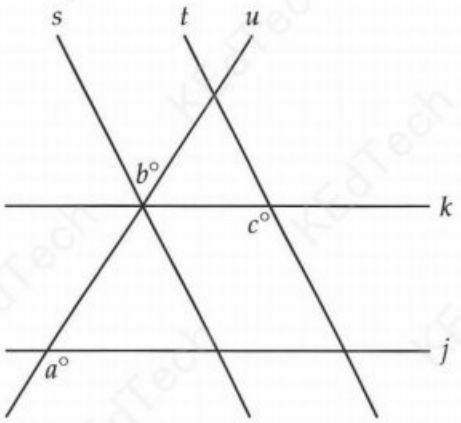
12



Note: Figure not drawn to scale.

In the figure, lines l and m are parallel. What is the value of x ?

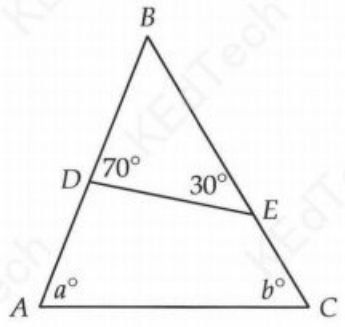
13



Note: Figure not drawn to scale.

In the figure, line u intersects parallel lines k and j and parallel lines s and t. If $a = 118$ and $b = 52$, what is the value of c ?

14

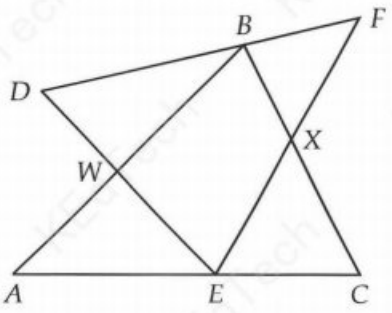


Note: Figure not drawn to scale.

In the figure, point D lies on $\overline{AB}$, and point E lies on $\overline{BC}$. What is the value of $a + b$?

A) 80
B) 100
C) 110
D) 120

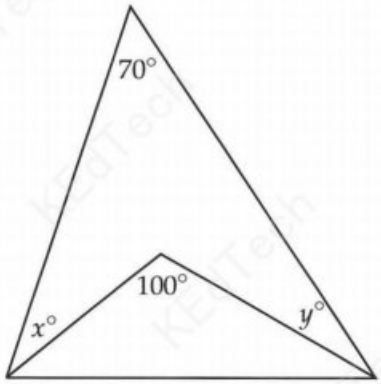
15



Note: Figure not drawn to scale.

In the figure shown, point B lies on line segment DF, point E lies on line segment AC, line segments AB and DE intersect at point W, and line segments BC and EF intersect at point X. The measure of $\angle A$ is 40°, the measure of $\angle C$ is 60°, the measure of $\angle D$ is 68°, and the measure of $\angle FBC$ is 72°. What is the measure, in degrees, of $\angle AED$?

16



Note: Figure not drawn to scale.

In the figure, two triangles share a common side. What is the value of $x + y$?

A) 10
B) 20
C) 30
D) 50