14

Inequalities

Just as we had equations and systems of equations, we can have inequalities and systems of inequalities.

The only difference is that you must reverse the sign every time you either multiply or divide both sides by a negative number.

For example, let's say we're trying to isolate x in the following inequality:

$$2x + 3 < 9$$

Do we have to reverse the sign at any point? Well, the first step is to subtract both sides by 3 to get $2x < 6$. Now we can divide both sides by 2 to get $x < 3$. At no point during this process did we multiply or divide by a negative number. Therefore, the sign stays the same.

Let's look at another example:

$$3x + 5 < 4x + 4$$

The first step is to combine like terms. We can subtract both sides by $4x$ to get the x's on the left-hand side. Then we can subtract both sides by 5 to get the constants on the right-hand side:

$$3x - 4x < 4 - 5$$
$$-x < -1$$

Notice that the sign hasn't changed yet. Now, to get rid of the negative in front of the x, we need to **multiply** both sides by -1. Doing so requires that we reverse the sign:

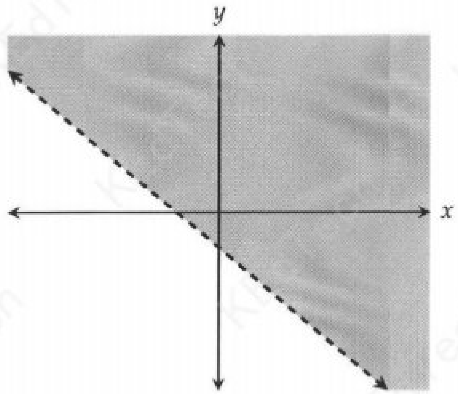
$$x > 1$$

Negative numbers in inequalities are the source of many mistakes, so let's be clear. Just working with negative numbers does NOT mean you need to change the sign. Some students see that they're dividing a negative number and impulsively reverse the sign. Don't do that. Only reverse the sign when you multiply or divide both sides **by** a negative number.

Graphs of inequalities

Most of the inequality questions on the SAT will not test your ability to solve inequalities. Rather, they'll test your understanding of how they're represented in the xy-plane.

Any inequality expressed in terms of x and y can be graphed in the xy-plane. For example, the inequality $y > -x - 1$ has the following graph:

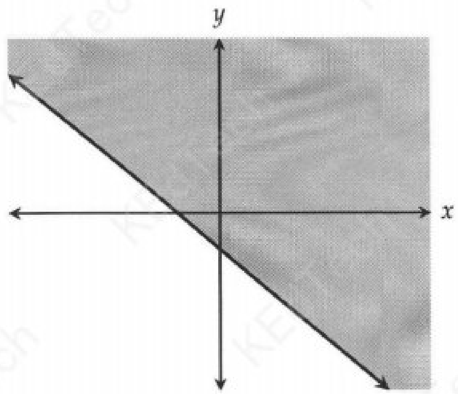


As you can see, the graph consists of the dashed line $y = -x - 1$ and a shaded region. Since the inequality we graphed is $y > -x - 1$, it makes sense that the shaded region consists of all the points above the line $y = -x - 1$. The shaded region also represents all the solutions to the inequality. In other words, any point in the shaded region (e.g. $(0,0)$) satisfies $y > -x - 1$.

If the sign in the inequality were reversed, the shaded region would lie below the line $y = -x - 1$, and any point within that shaded region would be a solution to $y < -x - 1$.

If you ever have a hard time determining whether a region is above or below a line, use the y-axis as a reference or draw a vertical line through the graph yourself if the y-axis isn't shown. For any line that cuts across the y-axis (or your vertical line), the top part of the y-axis is always in the "above" region, and the bottom part of the y-axis is always in the "below" region.

Lastly, we mentioned that the line is dashed. The dashed line indicates that the points on the line $y = -x - 1$ itself do not satisfy the inequality. If the inequality were $y \geq -x - 1$ instead of $y > -x - 1$, then the line in the graph would be solid, indicating that the points on the line do satisfy the inequality:



EXAMPLE 1:

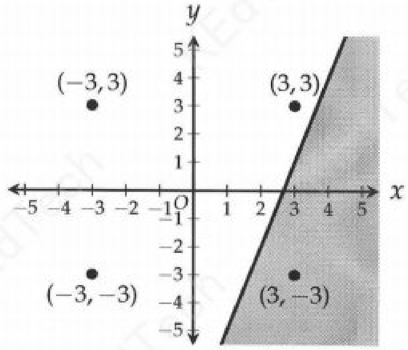
$$y \leq 3x - 8$$

Which point (x, y) is a solution to the given inequality in the xy-plane?

A) $(-3,-3)$ B) $(-3,3)$ C) $(3,-3)$ D) $(3,3)$

Test each point and see which one satisfies the inequality. Plugging in $(-3,-3)$ yields $-3 \leq 3(-3) - 8 \Rightarrow -3 \leq -17$, a false statement. Plugging in $(-3,3)$ yields $3 \leq 3(-3) - 8 \Rightarrow 3 \leq -17$, another false statement. Plugging in $(3,-3)$ yields $-3 \leq 3(3) - 8 \Rightarrow -3 \leq 1$, the only true statement. Answer $\boxed{\text{(C)}}$.

Graphing the inequality from this example, we get



Notice that the point $(3,-3)$ is the only answer choice that's inside the shaded solution region.

Compound inequalities

Compound inequalities are two or more inequalities that are combined to place more than one condition on the same single variable. For example, the compound inequality $2 \leq 5x \leq 3$ consists of $2 \leq 5x$ and $5x \leq 3$. When solving a compound inequality with three parts, any mathematical operations we perform must be applied to all three parts of the inequality.

EXAMPLE 2: If $-7 \leq -2x + 3 \leq 15$, which of the following must be true?

A) $5 \leq x \leq 6$ B) $-6 \leq x \leq -5$ C) $-6 \leq x \leq 5$ D) $-5 \leq x \leq 6$

To isolate x, we first subtract 3 from all three parts:

$$-7 - 3 \leq -2x + 3 - 3 \leq 15 - 3$$
$$-10 \leq -2x \leq 12$$

Now we divide everything by -2, reversing the signs:

$$\frac{-10}{-2} \geq \frac{-2x}{-2} \geq \frac{12}{-2}$$
$$5 \geq x \geq -6$$

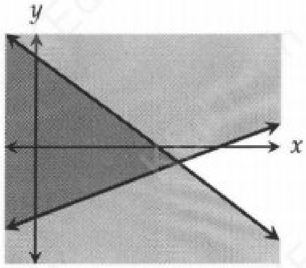
Since $5 \geq x \geq -6$ is equivalent to $-6 \leq x \leq 5$, the answer is $\boxed{\text{(C)}}$.

Systems of inequalities

To graph a system of inequalities, graph each inequality in the system one by one, shading in their individual solution regions. The solution region of the system is where all the individual shaded regions overlap. Let's look at the following example:

$$y \leq -x + 4$$
$$y \geq \frac{1}{2}x - 3$$

We need to shade the region below $y = -x + 4$ and the region above $y = \frac{1}{2}x - 3$.



The darkened overlapping region on the left contains all the points that are solutions to the system. These are the points that satisfy both inequalities in the system, not just one.

EXAMPLE 3:

$$x \leq 1$$
$$y \geq -2x + 1$$

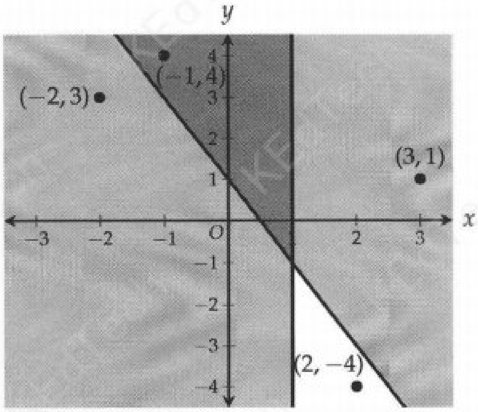
Which point (x, y) is a solution to the given system of inequalities in the xy-plane?

A) $(-2, 3)$ B) $(-1, 4)$ C) $(2, -4)$ D) $(3, 1)$

Let's plug in each point to see whether it satisfies both inequalities in the system.

The point $(-2, 3)$ yields $-2 \leq 1$, a true statement, from the first inequality and $3 \geq -2(-2) + 1 \Rightarrow 3 \geq 5$, a false statement, from the second inequality.

The point $(-1, 4)$ yields $-1 \leq 1$, a true statement, from the first inequality and $4 \geq -2(-1) + 1 \Rightarrow 4 \geq 3$, a true statement, from the second inequality. Since both statements are true, $(-1, 4)$ satisfies the system, and we can stop here. Answer $\boxed{(B)}$. To check our answer, let's graph the system of inequalities from this example:



As you can see, the only point from the answer choices that lies inside the overlapping solution region is $(-1, 4)$.

EXAMPLE 4:

$$y \leq 17$$
$$3x - 2y \leq -10$$

In the xy-plane, the point $(5, y)$ is a solution to the given system of inequalities. Which of the following could be the value of y ?

A) 6 B) 11 C) 14 D) 18

According to the first inequality, y can be no greater than 17. Since the second inequality contains an x term, we can plug the given point into the inequality to obtain more information about y:

$$3(5) - 2y \leq -10$$
$$-2y \leq -25$$
$$y \geq 12.5$$

So y must be at least 12.5 but no greater than 17. The only answer choice within this range is 14. Answer $\boxed{(C)}$.

Inequality word problems

EXAMPLE 5: James used no more than 400 grams of sugar to make chocolate chip cookies and shortbread cookies. He used 5 grams of sugar for each chocolate chip cookie and 7 grams of sugar for each shortbread cookie. Which of the following inequalities represents the possible numbers of chocolate chip cookies c and shortbread cookies s James could have made?

A) $\frac{5}{c} + \frac{7}{s} < 400$ B) $\frac{5}{c} + \frac{7}{s} \leq 400$ C) $5c + 7s < 400$ D) $5c + 7s \leq 400$

James used a total of $5c$ grams of sugar for the chocolate chip cookies and a total of $7s$ grams of sugar for the shortbread cookies. Altogether, he used $5c + 7s$ grams of sugar, an amount that must be less than or equal to 400 grams. Therefore, $5c + 7s \leq 400$. Answer $\boxed{(D)}$.

EXAMPLE 6: Hudson can work no more than 30 hours each week as a security guard. He wants to earn at least $1,800 each week working x regular hours and y overtime hours. If he earns $50 per regular hour and $80 per overtime hour, which of the following inequalities represents this situation?

A) $x + y \leq 30$
$50x + 80y \geq 1{,}800$

B) $x + y \leq 30$
$80x + 50y \geq 1{,}800$

C) $x + y \geq 30$
$50x + 80y \geq 1{,}800$

D) $x + y \geq 30$
$80x + 50y \geq 1{,}800$

Since Hudson can't work more than 30 hours each week, $x + y \leq 30$. The amount he earns from his regular hours is $50x$ and the amount he earns from his overtime hours is $80y$. Since Hudson wants his total earnings to be at least $1,800, $50x + 80y \geq 1{,}800$. Answer $\boxed{(A)}$.

Minimum & Maximum word problems

EXAMPLE 7: Corinne is a graphic designer who earns \$275 for every logo she designs. What is the minimum number of logos she would have to design to earn at least \$4,000 ?

Intuitive Approach: To earn at least \$4,000, Corinne would have to design at least $\frac{4{,}000}{275} \approx 14.5$ logos. That's 14 logos and half a logo. We have to assume, however, that a fraction of a logo cannot be designed and sold, so we must round up to $\boxed{15}$ logos.

Inequality Approach: If we let x be the number of logos Corinne designs, we can construct and solve the following inequality:

$$275x \geq 4{,}000$$
$$x \geq \frac{4{,}000}{275}$$
$$x \geq 14.5$$

The smallest whole number that satisfies this inequality is $\boxed{15}$.

When a whole number answer is implied, finding the minimum requires that we round up from a decimal answer. Finding the maximum requires that we round down from a decimal answer.

EXAMPLE 8: If one tray of flatbread can be made from 8 cups of flour and 6 cups of yogurt, what is the maximum number of whole trays of flatbread that can be made from 150 cups of flour and 100 cups of yogurt?

In these types of questions, one of the resources (e.g. either flour or yogurt) will be used up before the other, and that limiting resource will determine the amount of output that can be produced. To determine which resource is the limiting one, we have to consider each resource separately.

If we consider only the flour requirement, $\frac{150}{8} = 18.75$ trays of flatbread can be made. If we consider only the yogurt requirement, $\frac{100}{6} \approx 16.7$ trays of flatbread can be made. Since the amount that can be produced from the yogurt is less than the amount that can be produced from the flour, the yogurt is the **limiting factor**. There isn't enough of it to use up the flour, so we're limited to 16.7 trays of flatbread. As a result, the maximum whole number of trays that can be made is $\boxed{16}$. Remember that we round down from a decimal answer when finding the maximum.

EXAMPLE 9: An art teacher needs to buy a total of 36 paintbrushes for a painting class. Each paintbrush must be either an acrylic brush, which costs \$5, or a watercolor brush, which costs \$3. If no more than \$150 can be spent on the paintbrushes, what is the minimum number of watercolor brushes the art teacher can buy?

Let a be the number of acrylic brushes and w be the number of watercolor brushes. Now we can set up a system consisting of an equation and an inequality:

$$\begin{aligned} a + w &= 36 \\ 5a + 3w &\leq 150 \end{aligned}$$

The goal now is to get a single inequality that we can solve for w. The first step is to isolate a in the first equation to get $a = 36 - w$. Now we can substitute $36 - w$ for a in the inequality:

$$\begin{aligned} 5(36 - w) + 3w &\leq 150 \\ 180 - 5w + 3w &\leq 150 \\ 180 - 2w &\leq 150 \\ -2w &\leq -30 \\ w &\geq 15 \end{aligned}$$

Based on this result, the minimum number of watercolor brushes that can be bought is $\boxed{15}$.

EXAMPLE 10: Ava is decorating two-tier and three-tier wedding cakes. It takes her 20 minutes to decorate each two-tier wedding cake and 35 minutes to decorate each three-tier wedding cake. If Ava needs to decorate at least 14 wedding cakes today, and she can spend no more than 6 hours doing so, what is the maximum number of three-tier wedding cakes she could decorate today?

Let x be the number of two-tier cakes and y be the number of three-tier cakes Ava decorates. Now we can set up the following system:

$$\begin{aligned} 20x + 35y &\leq 360 \\ x + y &\geq 14 \end{aligned}$$

Note that we converted 6 hours to 360 minutes to set up the first inequality, which we can reduce to get the following system:

$$\begin{aligned} 4x + 7y &\leq 72 \\ x + y &\geq 14 \end{aligned}$$

Since we have a system of two inequalities, we can't just do what we did in Example 8, which gave us a system with one equation and one inequality. Instead, we have to use one of the following approaches:

Approach 1: The maximum possible value of y (i.e. maximum number of three-tier wedding cakes) can be found by minimizing x (number of two-tier wedding cakes). Since the second inequality in the system gives $x \geq 14 - y$, the minimum possible value of x is equivalent to $14 - y$. Substituting this result into the first inequality, we get

$$\begin{aligned} 4(14 - y) + 7y &\leq 72 \\ 56 - 4y + 7y &\leq 72 \\ 3y &\leq 16 \\ y &\leq 5.33\ldots \end{aligned}$$

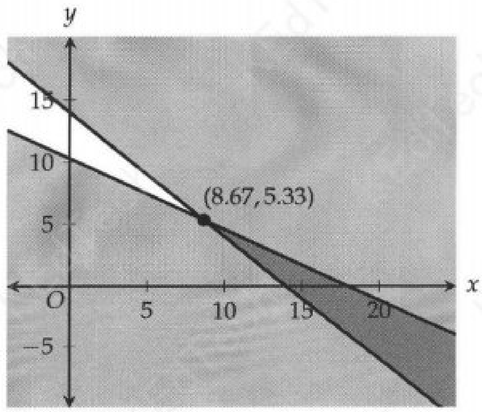
The maximum possible value of y is then $\boxed{5}$ (since it's implied that Ava decorates cakes in whole numbers).

Approach 2: Inequalities can be added together if their signs point in the same direction. Note that inequalities should never be subtracted from one another; think only in terms of addition. In this case, we can multiply the second inequality by -4 to align both the signs and the coefficients of x:

$$4x + 7y \leq 72$$
$$-4x - 4y \leq -56$$

Remember that multiplying by a negative number reverses the sign. Now we can add these two inequalities together to get $3y \leq 16$, which simplifies to $y \leq 5.33$. Based on this result, the maximum possible (whole number) value of y is $\boxed{5}$.

Approach 3 (best): We can graph the system of inequalities on Desmos and look for the point with the maximum value of y in the overlapping solution region (the highest point in the region).



The highest point in the overlapping solution region turns out to be (8.67, 5.33), which indicates that $y \leq 5.33$. The maximum possible (whole number) value of y is then $\boxed{5}$.

EXERCISE 1: Answers for this exercise start on page 366.

1

$$y > 7x - 19$$

Which of the following points in the xy-plane satisfies the given inequality?

A) $(-1, -32)$
B) $(0, -20)$
C) $(2, -3)$
D) $(4, -14)$

2

Katherine has 28 classroom calculators that each require a set of 4 batteries. If her school supplies her with batteries in packs of 6, what is the least number of packs needed to provide every classroom calculator with a complete set of batteries?

3

$$y < -4x - 1$$
$$y > x + 2$$

Which point (x, y) is a solution to the given system of inequalities in the xy-plane?

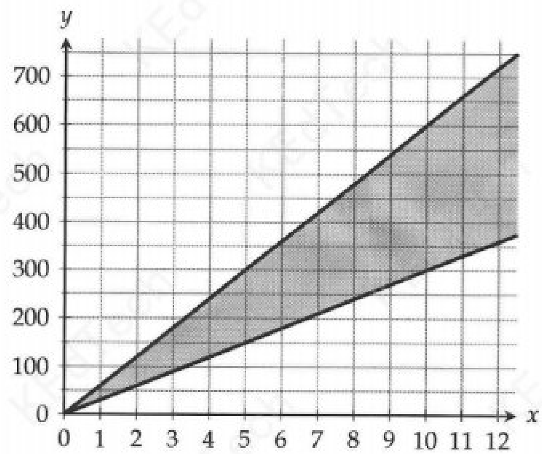
A) $(1,4)$
B) $(-1,1)$
C) $(-2,2)$
D) $(-3,-3)$

4

A hot dog stand sells more than \$800 worth of hot dogs each day. The price of a hot dog is \$3 during lunch and \$5 during dinner. Let l represent the number of hot dogs sold during lunch yesterday, and let d represent the number of hot dogs sold during dinner yesterday. Which of the following inequalities represents all possible values of l and d ?

A) $3l + 5d > 800$
B) $3l + 5d \geq 800$
C) $5l + 3d > 800$
D) $5l + 3d \geq 800$

5



A train travels at a speed of at least 30 miles per hour and no more than 60 miles per hour. The graph shows the possible distances y, in miles, the train has traveled after x hours. Which ordered pair (x, y) represents a possible distance y, in miles, the train has traveled after x hours?

A) $(4,375)$
B) $(6,450)$
C) $(9,250)$
D) $(11,625)$

6

In the xy-plane, the solution region of an inequality contains the points $(2, 2)$, $(-1, -1)$, and $(-2, -4)$. Which of the following could be the inequality?

A) $y \leq \frac{3}{2}x - 1$

B) $y \geq \frac{3}{2}x - 1$

C) $y \leq -3x + 2$

D) $y \geq -3x + 2$

7

During a week-long fishing trip, Ashleigh caught nine less than three times the number of fish Naomi caught. If they caught at least 45 fish combined, what is the minimum number of fish that Naomi could have caught?

8

To restock supplies, a nail salon purchases toolkits that each include 80 nail files and 150 nail buffers. If the nail salon needs to restock at least 1,800 nail files and at least 4,000 nail buffers, what is the minimum number of toolkits the salon must purchase?

9

$$x + y < 25$$
$$2x - 3y < 77$$

In the xy-plane, the point $(18, y)$ is a solution to the given system of inequalities. Which of the following could be the value of y ?

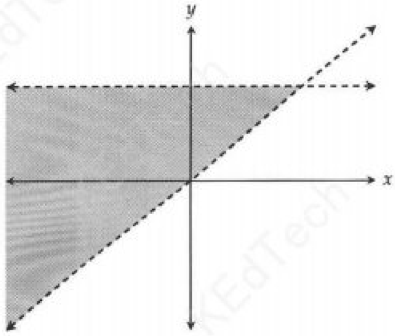
A) -14

B) -6

C) 7

D) 11

10



The shaded region shown could represent the solutions to which of the following systems of inequalities?

A) $y > 3$
$y > x$

B) $y < 3$
$y < x$

C) $y < 3$
$y > x$

D) $y > 3$
$y < x$

11

To get to work each day, Harry must travel 8 miles by bus and 16 miles by train. The bus travels at an average speed of x miles per hour and the train travels at an average speed of y miles per hour. If Harry's daily commute never takes more than 1 hour, which of the following inequalities represents the possible average speeds of the bus and train during the commute?

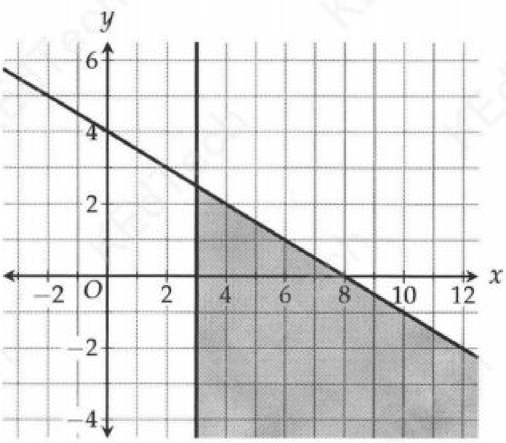
A) $\frac{8}{x} + \frac{16}{y} \leq 1$

B) $\frac{16}{x} + \frac{8}{y} \leq 1$

C) $\frac{x}{8} + \frac{y}{16} \leq 1$

D) $8x + 16y \leq 1$

12



The shaded region of the graph represents the solution to which of the following system of inequalities?

A) $y \geq 3$
$y \leq -2x + 4$

B) $y \geq 3$
$y \geq -2x + 4$

C) $x \geq 3$
$2y \leq -x + 8$

D) $x \geq 3$
$2y \geq -x + 8$

13

Each month, Eric spends 40% of his monthly salary and contributes the rest to his investment account. His goal is to contribute at least \$50,000 to his investment account each year. Which of the following is the closest to the minimum possible monthly salary Eric needs to earn to achieve his goal each year?

A) \$5,788

B) \$6,945

C) \$8,334

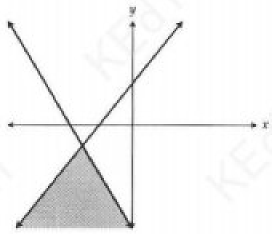
D) \$10,417

14

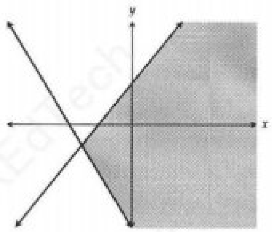
$$y \geq \frac{3}{2}x + 2$$
$$y \leq -2x - 5$$

Which of the following graphs in the xy-plane could represent the given system of inequalities?

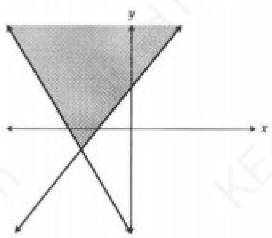
A)



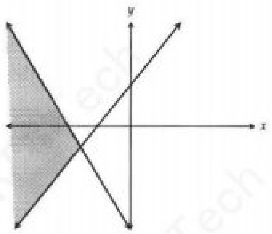
B)



C)



D)



15

Tina works no more than 30 hours at a nail salon each week. She can do a manicure in 20 minutes and a pedicure in 30 minutes. Each manicure earns her \$25 and each pedicure earns her \$40, and she must earn at least \$900 to cover her expenses. If m represents the number of manicures Tina does in a week and p represents the number of pedicures she does in a week, which system of inequalities describes all possible values of m and p for which Tina covers her expenses?

A) $3m + 2p \leq 30$
$25m + 40p \geq 900$

B) $2m + 3p \leq 30$
$25m + 40p \geq 900$

C) $\frac{m}{3} + \frac{p}{2} \leq 30$
$25m + 40p \geq 900$

D) $\frac{m}{3} + \frac{p}{2} \geq 900$
$25m + 40p \leq 30$

16

During hibernation, adult male black bears lose 6 to 12 pounds in weight each week. An adult male black bear weighed 580 pounds after 5 weeks of hibernation. Which of the following inequalities gives all possible values for the black bear's weight w, in pounds, after 15 weeks of hibernation?

A) $400 \leq w \leq 490$
B) $460 \leq w \leq 520$
C) $460 \leq w \leq 580$
D) $610 \leq w \leq 640$

17

Manasa uses an automated system that pays her taxes on the day they are due. The system will pay 25% of her taxes from her checking account and the remainder of her taxes from her savings account. If there is \$3,300 in Manasa's checking account and \$4,500 in her savings account on the day her taxes are due, which inequality gives the amount in taxes t, in dollars, for which her checking account will have a higher balance than her savings account after the taxes are paid?

A) $t < 2{,}400$
B) $t > 2{,}400$
C) $t < 2{,}700$
D) $t > 2{,}700$

18

A manufacturer ships only 3-pound and 5-pound rugs to its stores. Each shipment must contain at least 85 rugs, and the total weight of the rugs can be no more than 400 pounds. What is the greatest possible number of 5-pound rugs each shipment can contain?

19

Katy needs to cover an area of at least 800 square feet using ceramic and porcelain tiles. Each ceramic tile costs \$4.70 and covers an area of 4 square feet. Each porcelain tile costs \$15.20 and covers an area of 7 square feet. If Katy has a budget of \$1,600 to spend on these tiles, what is the minimum number of ceramic tiles she can purchase?

EXERCISE 2: Answers for this exercise start on page 368.

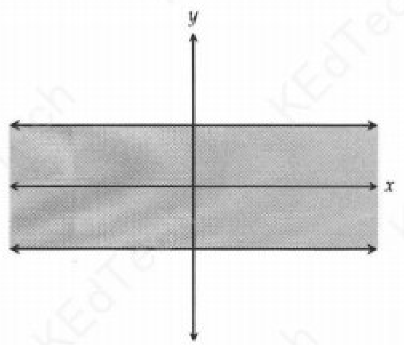
1

$$y > 3x - 20$$
$$y < \frac{1}{2}x + 5$$

Which point (x, y) is a solution to the given system of inequalities in the xy-plane?

A) $(-10, 0)$
B) $(10, 0)$
C) $(10, 10)$
D) $(-10, -10)$

2



The shaded region of the graph could represent the solutions to which of the following systems of inequalities?

A) $y \geq 3$
$y \leq -3$

B) $y \leq 3$
$y \geq -3$

C) $x \geq 3$
$x \leq -3$

D) $x \leq 3$
$x \geq -3$

3

A gift shop held a weekend sale with the goal of selling at least \$8,000 worth of greeting cards and gift boxes. Each greeting card was sold for \$5, and each gift box was sold for \$7. If 400 gift boxes were sold during the sale, what is the minimum number of greeting cards the shop could have sold to meet its goal?

A) 1,040
B) 1,160
C) 1,280
D) 1,400

4

The point $(5, 0)$ in the xy-plane is a solution to which of the following systems of inequalities?

A) $y > x$
$y > -x$

B) $y > x$
$y < -x$

C) $y < x$
$y > -x$

D) $y < x$
$y < -x$

5

For an inequality graphed in the xy-plane, the points $(1, 4)$ and $(3, 0)$ are contained in the solution region, and the point $(1, -2)$ is not contained in the solution region. Which of the following could be the inequality?

A) $y > -2x + 1$
B) $y \geq x - 4$
C) $x < 2$
D) $y \leq -3x + 7$

6

An ice cream distributor contracts out to two different companies to manufacture cartons of ice cream. Company A can produce 80 cartons per hour and Company B can produce 140 cartons per hour. To fulfill an order of over 1,100 cartons in 10 total hours of contract time, the distributor contracts out x hours to Company A and the remaining hours to Company B. Which of the following inequalities gives all possible values of x in the context of this problem?

A) $\dfrac{80}{x} + \dfrac{140}{10-x} > 1{,}100$

B) $140x + 80(10 - x) > 1{,}100$

C) $80x + 140(10 - x) > 1{,}100$

D) $80x + 140(x - 10) > 1{,}100$

7

Melanie burns 120 to 135 calories for each mile that she jogs. Which inequality represents the total number of calories, y, Melanie could burn from jogging 14 miles?

A) $120 \leq y + 14 \leq 135$

B) $120 \leq y - 14 \leq 135$

C) $120 \leq 14y \leq 135$

D) $120 \leq \dfrac{y}{14} \leq 135$

8

A pharmacy produces a certain medication in a daytime variety and a nighttime variety. A bottle of the daytime variety contains 2 ounces of the active ingredient and 6 ounces of flavored syrup. A bottle of the nighttime variety contains 3 ounces of the active ingredient and 5 ounces of flavored syrup. The pharmacy currently has no more than 385 ounces of the active ingredient and no more than 850 ounces of flavored syrup available. If at least 65 bottles of the daytime variety must be filled, what is the maximum number of bottles of the nighttime variety that can be filled?

A) 78

B) 85

C) 92

D) 106

9

Denise is given a \$20 bill to buy 45-cent stamps and 60-cent stamps at the post office. She is told that she must return with at least \$5 in change. Which of the following inequalities represents this situation, where x is the number of 45-cent stamps Denise can buy, and y is the number of 60-cent stamps Denise can buy? (Assume there is no sales tax.)

A) $0.45x + 0.60y \geq 5$

B) $0.45x - 0.60y \leq 5$

C) $0.45x + 0.60y \geq 15$

D) $0.45x + 0.60y \leq 15$

10

The number k is 7 more than half the maximum value of y. Which inequality shows the possible values of y ?

A) $y \leq 2k - 7$

B) $y \geq 2k - 7$

C) $y \leq 2k - 14$

D) $y \geq 2k - 14$

11

Sandra is given a budget of \$250 to buy pencils and enough pencil cases to hold all of them. Each pencil case costs \$4.50 and holds up to 12 pencils. Each pencil costs \$1.50. Which of the following systems of inequalities represents this situation in terms of the number of pencils Sandra can buy, x, and the number of pencil cases she can buy, y ?

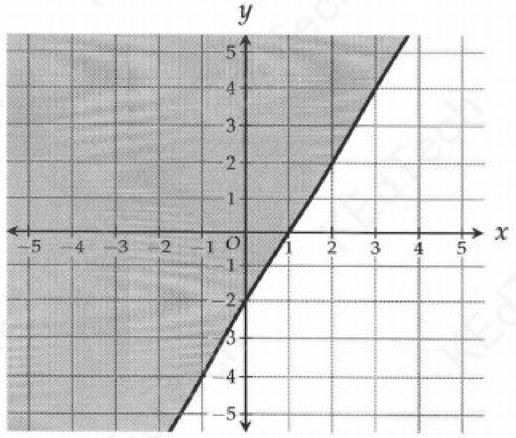
A) $1.50x + 4.50y \leq 250$
$y \geq 12x$

B) $1.50x + 4.50y \geq 250$
$y \leq 12x$

C) $1.50x + 4.50y \leq 250$
$y \geq \dfrac{x}{12}$

D) $1.50x + 4.50y \geq 250$
$y \geq \dfrac{x}{12}$

12



The solutions to which of the following inequalities are represented by the shaded region of the graph?

A) $x - 2y \leq 4$

B) $x - 2y \geq 4$

C) $2x - y \leq 2$

D) $2x - y \geq 2$

13

A tool manufacturer wants to produce a wrench set that consists of a toolbox and 7 wrenches. The toolbox costs t dollars to produce and each wrench costs w dollars to produce. The total cost of producing the wrench set is at least 4 times but no more than 6 times the cost of producing the toolbox. Which inequality represents all possible values of w ?

A) $\frac{3}{7}t \leq w \leq \frac{5}{7}t$

B) $3t \leq w \leq 5t$

C) $\frac{4}{7}t \leq w \leq \frac{6}{7}t$

D) $4t \leq w \leq 6t$

14

Paul wants to purchase and install 30 new home windows. The installation fee on each window is $50 plus 10% of the price of each window. The most Paul can spend on the windows, including the installation fees, is $25,000. Which of the following is closest to the maximum possible price per window, before the installation fee, Paul could pay?

A) $712.12

B) $756.06

C) $783.33

D) $828.90

15

Jerry estimates that there are m marbles in a jar. Harry, who knows the actual number of marbles in the jar, notes that the actual number, n, is within 10 marbles (inclusive) of Jerry's estimate. Which of the following inequalities represents the relationship between Jerry's estimate and the actual number of marbles in the jar?

A) $n + 10 \leq m \leq n - 10$

B) $m - 10 \leq n \leq m + 10$

C) $n \leq m \leq 10n$

D) $\frac{m}{10} \leq n \leq 10m$

16

$$y > 15x + a$$
$$y < 5x + b$$

In the given system of inequalities, a and b are constants. If the point $(1, 20)$ is a solution to the system in the xy-plane, which of the following could be the value of $b - a$?

A) 6

B) 8

C) 10

D) 12

17

John makes cedar and oak birdhouses to sell in his shop. To fill his shop's showroom, he must make at least 75 birdhouses. Each cedar birdhouse takes 165 minutes to make and each oak birdhouse takes 75 minutes to make. If John can spend at most 160 hours making birdhouses for his showroom, what is the maximum number of cedar birdhouses he could make?

18

A school play had 240 audience members consisting of adults and children. There was at least one adult for every 2 children. Which of the following inequalities gives the possible values of a, the number of adults in the audience?

A) $a > 80$

B) $a \geq 80$

C) $a > 120$

D) $a \geq 120$

19

A barista wants to use Arabica and Robusta beans to make a cup of coffee containing less than 280 milligrams of caffeine. Each ounce of Arabica beans contains 40 milligrams of caffeine and each ounce of Robusta beans contains 60 milligrams of caffeine. If the barista is required to use twice as many Arabica beans as Robusta beans, which inequality gives all possible values for the amount a, in ounces, of Arabica beans the barista could use?

A) $0 < a < 1\frac{3}{4}$

B) $0 < a < 2$

C) $0 < a < 3\frac{1}{2}$

D) $0 < a < 4$

20

During each of her workouts, Mina burns at least 700 calories from walking and running, and she runs for at least 20 minutes longer than she walks. Mina burns 5.8 calories per minute while walking and 10.2 calories per minute while running. What is the minimum amount of time, in minutes, she could spend running during each workout?

21

Lianne wants to make a seasoning that consists of 75% sea salt and 25% black pepper. If sea salt costs \$2 per pound and black pepper costs \$8 per pound, and Lianne can spend no more than \$210 on these ingredients, what is the maximum number of pounds of seasoning that she will be able to make?

A) 42

B) 50

C) 56

D) 60